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Types of Separation Axioms In Micro Bitopological Space

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ABSTRACT

In this paper, we introduced the different type of separation axioms in micro bitopological space, like $M\mu_{R_{1,2}} - T_i$ sets and $M\mu_{R_{1,2}} - D_i$ sets where $i = 0,1,2$ and properties with some examples. We also discuss symmetric on micro bitopological space. We talked those properties also.

Keywords: *Bitopological space, Separation axioms, Symmetric, Difference set.*

1. INTRODUCTION

"Toloplogy" is a relatively new field of mathematics, with the majority of its research conducted since 1900. Properties of spaces that remain unchanged during continuous deformations are studied in topology. Because the objects may be stretched and contracted like rubber yet cannot be broken, it is frequently referred to as "rubber-sheet geometry". Bitopology origin is usually associated with the appearance of J. C. Kelly's [1] paper in 1963. In this fundamental paper, a bitopological space was clearly defined as a set with two arbitrary topological structure.

M. Chiangpradit, S. Sompong, C. Boonpok [2] where define the defintion for separation axioms on bitopological spaces and give some examples and N. Chutiman, S. Spmpong, C. Boonpok [3], In that paper they where introducted differential set and separation axioms for that set. We discover a definition of micro bitopological space and different types of micro open sets[4].

We introduced differnet type of separation axioms in micro bitopological space with some features and an example in the current study. By using examples, we were able to teach several types of $M\mu_{R_{1,2}} - T_i$ sets and compare them. Additionally, we provided instances of several types of $M\mu_{R_{1,2}} - D_i$ sets. The notion of $M\mu_{R_{1,2}} -$ symmetric set was covered.

2. PRELIMINARY

Definition 2.1 [1]

Let X be a set. Let P and Q be topologies for X. Then the ordered triple (X, P, Q) is said

to be a bitopological space.

Definition 2.2 [2]

A bitopological space (X, τ_1, τ_2) is said to be: (1) $(\tau_1, \tau_2) - T_0$ if for any pair of distinct points in X there exists a $\tau_1\tau_2 -$ open set containing one of the points not the other. (2) $(\tau_1, \tau_2) - T_1$ if for any pair of distinct points $x, y \in X$, there exist $\tau_1\tau_2 -$ open sets U and V such that $x \in U, y \notin U$ and $y \in V, x \notin V$.

Definition 2.3 [3]

A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2) - T_2$ if for any pair of distinct points $x, y \in X$, there exist disjoint $\tau_1\tau_2 -$ open sets U and V of X containing x and y, respectively.

Definition 2.4 [3]

A bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2) symmetric if for each $x, y \in X$, $x \in \tau_1\tau_2 - cl\{y\}$ implies $y \in \tau_1\tau_2 - cl\{x\}$.

Definition 2.5 [3]

A subset A of a bitopological space (X, τ_1, τ_2) is called a $(\tau_1, \tau_2) - D$ set if there exist $\tau_1\tau_2 -$ open set U and V of X such that $U \neq X$ and $A = U - V$.

Definition 2.6 [4]

Let U be a non empty set and it contain two equivalence relation R_1, R_2. Let X_1, X_2 be any two subsets of U. Let $(U, \tau_{R_1}(X_1))$ is a nano topological space then $\mu_{R_1}(X_1) = \{S \cup (S' \cap \mu_1) / S, S' \in \tau_{R_1}(X_1)\}$ where $\mu_1 \notin \tau_{R_1}(X_1)$ is called a micro topology of $\tau_{R_1}(X_1)$ and let $(U, \tau_{R_2}(X_2))$ is an another nano topological space then $\mu_{R_2}(X_2) = \{T \cup (T' \cap \mu_2) / T, T' \in \tau_{R_2}(X_2)\}$ where $\mu_2 \notin \tau_{R_2}(X_2)$ is called a micro topology of $\tau_{R_2}(X_2)$. If $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2), \mu_{R_1}(X_1), \mu_{R_2}(X_2))$ briefly, $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is called a micro bitopological space then it is satisfy the following conditions:

1. $U, \phi \in \mu_{R_{1,2}}(X)$.
2. Arbitrary union of $\mu_{R_{1,2}}(X)$ is in $\mu_{R_{1,2}}(X)$.
3. Finite intersection of $\mu_{R_{1,2}}(X)$ is in $\mu_{R_{1,2}}(X)$.

Therefore $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is called a micro bitopological space and the element of micro bitopological space is called $M\mu_{R_{1,2}}(X)-$ open. The complement is called $M\mu_{R_{1,2}}(X)-$ closed set.

Definition 2.7 [4]

Let H be a subset of a micro bitopological space. Then the micro closure of H is denoted by

$M\mu_{R_{1,2}}cl(H) = \cap \{ V : H \subseteq V \}$ and V is $M\mu_{R_{1,2}}(X)$ - closed. Then the micro interior of H is

denoted by $M\mu_{R_{1,2}}int(H) = \cup \{ V : V \subseteq H \}$ and V is $M\mu_{R_{1,2}}(X)$- open.

3. MAIN RESULTS

Definition 3.1

A micro bitopological space $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be $M\mu_{R_{1,2}} - T_0$ if there is any two distinct points in U there exists a $M\mu_{R_{1,2}} -$ open set of U which includes anyone of them but not the other.

Definition 3.2

A micro bitopological space $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be $M\mu_{R_{1,2}} - T_1$ if there is any two distinct points $l, m \in U$ there exists $M\mu_{R_1}$ open set A and $M\mu_{R_2}$ open set B of U which includes anyone of them but

not the other.

Definition 3.3

A micro bitopological space $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be $M\mu_{R_{1,2}} - T_2$ if there are any two distinct points $l, m \in U$, there exists a disjoint $M\mu_{R_1}$ open set A and $M\mu_{R_2}$ open set B of U containing l and m respectively.

Example 3.4

Let $U = \{1, 2, 3\}$ with $U/R_1 = \{\{1\}, \{2, 3\}\}$ and $X_1 = \{1\}$, $\tau_{R_1}(X_1) = \{U, \phi, \{1\}\}$ Then $\mu_1 = \{2\} \notin \tau_{R_1}(X_1)$. The $M\mu_{R_1}(X_1) = \{U, \phi, \{1\}, \{2\}, \{1, 2\}\}$ and $U/R_2 = \{\{2\}, \{1, 3\}\}$ and $X_2 = \{1,2\} \Longrightarrow \tau_{R_2}(X_2) = \{U, \phi, \{2\}, \{1, 3\}\}$. Then $\mu_2 = \{3\} \notin \tau_{R_2}(X_2)$.Then the $M\mu_{R_2}(X_2) = \{U, \phi, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}\}$. Then the $M\mu_{R_{1,2}}(X) = \{U, \phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$ is a micro bitopological space. If take $1,2 \in U$ then $1 \in \{1\}$ and $2 \notin \{1\}$. Its $M\mu_{R_{1,2}} - T_0$ space.

In this same way we can prove that for U. Let $A = \{1, 2\}$ and $B = \{1, 3\}$. Take $2,3 \in U$. Then $2 \in A$, $2 \notin B$ and $3 \notin A$, $3 \in B$. Its $M\mu_{R_{1,2}} - T_1$ space. This example is also satisfy U is $M\mu_{R_{1,2}} - T_2$ space.

Definition 3.5

A micro bitopological space $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be $M\mu_{R_{1,2}} -$ Symmetric if for each $l, m \in U$, $l \in$

$M\mu_{R_{1,2}} cl\{m\}$ implies $m \in M\mu_{R_{1,2}} cl\{l\}$.

Example 3.6

Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a, b\}, \{c\}, \{d\}\}$ and $X_1 = \{b\}$, $\tau_{R_1}(X_1) = \{U, \phi, \{a, b\}\}$ Then $\mu_1 = \{a\} \notin \tau_{R_1}(X_1)$. The $M\mu_{R_1}(X_1) = \{U, \phi, \{a\}, \{a, b\}\}$ and $U/R_2 = \{\{a\}, \{b, c\}, \{d\}\}$ and $X_2 = \{a\} \implies \tau_{R_2}(X_2) = \{U, \phi, \{a\}\}$. Then $\mu_2 = \{a, b\} \notin \tau_{R_2}(X_2)$. Then the $M\mu_{R_2}(X_2) = \{U, \phi, \{a\}, \{a, b\}\}$. Then the $M\mu_{R_{1,2}}(X) = \{U, \phi, \{a\}, \{a, b\}\}$ is a micro bitopological space.

Take $c, d \in U$ it is $M\mu_{R_{1,2}} -$ Symmetric.

Theorem 3.7

A micro bitopological space is $M\mu_{R_{1,2}} - T_0$ space iff the $M\mu_{R_{1,2}}$ closure of any two distinct points of a micro bitopological space is not equal.

Proof

Assume that, micro closure of any two distinct points x, y of a micro bitopological space is not equal. Let x, y $\in U$ such that x $\in M\mu_{R_{1,2}} cl\{x\}$ and y $\in M\mu_{R_{1,2}} cl\{y\}$. We need to prove, U is $M\mu_{R_{1,2}} - T_0$ space. Let $l \in U$ such that $l \in M\mu_{R_{1,2}} cl\{x\}$ but $l \notin M\mu_{R_{1,2}} cl\{y\}$. Suppose $x \in M\mu_{R_{1,2}} cl\{y\}$. Then $M\mu_{R_{1,2}} cl\{x\} \subseteq M\mu_{R_{1,2}} cl\{y\}$ which is a contradiction that $l \notin M\mu_{R_{1,2}} cl\{y\}$. $x \in M\mu_{R_{1,2}}(X) - M\mu_{R_{1,2}} cl\{y\}$. Therefore, U is $M\mu_{R_{1,2}} - T_0$ space. Conversely, assume that U is $M\mu_{R_{1,2}} - T_0$ space. Let $x, y \in U$ then there exists a open set V its contain x but not y.

Then $U - V$ is $M\mu_{R_{1,2}}$ closed set containing y. Then the $M\mu_{R_{1,2}} cl\{y\}$ is the smallest closed set containing y, therefore $y \in M\mu_{R_{1,2}} cl\{y\} \subseteq U - V$. By assumption $x \notin M\mu_{R_{1,2}} cl\{y\}$. $M\mu_{R_{1,2}} cl\{x\} \neq M\mu_{R_{1,2}} cl\{y\}$.

Theorem 3.8

A micro bitopological space is $M\mu_{R_{1,2}} - T_1$ space iff every singleton set in a micro bitopological space is $M\mu_{R_{1,2}}$ closed.

Proof

Assume that every singleton set is $M\mu_{R_{1,2}}$ closed. Let $l \in U$ then $\{l\}$ is $M\mu_{R_{1,2}}$ closed. Let x, y $\in U, x \neq y$ then y $\in U - \{x\}$ is $M\mu_{R_{1,2}}$ open set containing y but not x. In the same way, x $\in U - \{y\}$ is also a $M\mu_{R_{1,2}}$ open set containing x but not y. Therefore U is $M\mu_{R_{1,2}} - T_1$ space. Conversely, assume that U is $M\mu_{R_{1,2}} - T_1$ space. To prove the singleton sets are $M\mu_{R_{1,2}}$ closed. Let x, y $\in U$, $x \neq y$. By assumption, y $\in U - \{x\}$ is $M\mu_{R_{1,2}}$ $-$open set. There is a $M\mu_{R_{1,2}}$ open set V containing y but not x . Therefore, $y \in V \subseteq U - \{x\}$. Then $V - \{x\}$ is micro open. Therefore $\{x\}$ is $M\mu_{R_{1,2}}$ closed. In this same way we can prove that for $\{y\}$ is $M\mu_{R_{1,2}}$ closed.

Theorem 3.9

A micro bitopological space is $M\mu_{R_{1,2}} -$ symetric space iff for each, $x \in U, M\mu_{R_{1,2}} cl(x) \subseteq X$ is $M\mu_{R_{1,2}} -$ open and $x \in X$.

Proof

Assume that, $M\mu_{R_{1,2}} cl(x) \subseteq X$ is $M\mu_{R_{1,2}} -$ open and $x \in X, x \in U$. To prove, $M\mu_{R_{1,2}} -$ symetric space. Suppose that $x \in M\mu_{R_{1,2}} cl(y)$ but $y \notin M\mu_{R_{1,2}} cl(x)$. Then y belongs to the complement of $M\mu_{R_{1,2}} cl(x)$. Already know that $y \in M\mu_{R_{1,2}} cl(y)$. $M\mu_{R_{1,2}} cl(y) \subseteq$ complement of $M\mu_{R_{1,2}} cl(x)$. Then we got, $x \in$ the complement of $M\mu_{R_{1,2}} cl(x)$. Which is a contradiction. Then $y \in M\mu_{R_{1,2}} cl(x)$. It is $M\mu_{R_{1,2}} -$ symetric space. Conversely, assume that $M\mu_{R_{1,2}} -$ symetric space. To prove, $x \in U, M\mu_{R_{1,2}} cl(x) \subseteq X$ is $M\mu_{R_{1,2}} -$ open and $x \in X$. Let $x \subseteq X$ is $M\mu_{R_{1,2}} -$ open. Suppose that $M\mu_{R_{1,2}} cl(x) \nsubseteq X$. Then $M\mu_{R_{1,2}} cl(x) \subseteq$ the complement of X. Then $M\mu_{R_{1,2}} cl(x) \cap c(X) \neq \phi$. Let $y \in M\mu_{R_{1,2}} cl(x) \cap c(X) \Rightarrow y \in M\mu_{R_{1,2}} cl(x)$. and $y \in c(X)$. Already know, $y \in M\mu_{R_{1,2}} cl(y)$. Therefore, $M\mu_{R_{1,2}} cl(y) \subseteq c(X)$. By given, $x \in M\mu_{R_{1,2}} cl(y)$. Then $x \in c(X)$ which is a contradiction. Therefore, $M\mu_{R_{1,2}} cl(x) \subseteq X$.

Theorem 3.10

To illustrate that $M\mu_{R_{1,2}} - T_0$ space is a hereditary property.

Proof

A micro bitopological space $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is a $M\mu_{R_{1,2}} - T_0$ space and $N \subseteq U$ then to prove that $(N, \tau_{R_{1,2}}(N), \mu_{R_{1,2}}(N))$ is also satisfy $M\mu_{R_{1,2}} - T_0$ axioms. Let $a, b \in N$ with $a \neq b$ then $a, b \in U$ with $a \neq b$. By our taken $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is a $M\mu_{R_{1,2}} - T_0$ space then there exists $S \in M\mu_{R_{1,2}}$ open set such that $a \in S, b \notin S$ or $b \in S, a \notin S$. $S \in M\mu_{R_{1,2}} \Rightarrow S \in M\mu_{R_1}$ or $S \in M\mu_{R_2} \Rightarrow S \cap N \in M\mu_{R_1}(N)$ or $S \cap N \in M\mu_{R_2}(N) \Rightarrow S \cap N \in M\mu_{R_{1,2}}(N)$. Again since, $a, b \in N$ then $a \in S \cap N, b \notin S \cap N$ or $b \in S \cap N, a \notin S \cap N$. Hence N satisfy.

Theorem 3.11

To illustrate that $M\mu_{R_{1,2}} - T_1$ space is a hereditary property.

Proof

A micro bitopological space $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is a $M\mu_{R_{1,2}} - T_1$ space and $N \subseteq U$ then to prove that $(N, \tau_{R_{1,2}}(N), \mu_{R_{1,2}}(N))$ is also satisfy $M\mu_{R_{1,2}} - T_1$ axioms. Let $a, b \in N$ with $a \neq b$ then $a, b \in U$ with $a \neq b$. By our taken $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is a $M\mu_{R_{1,2}} - T_1$ space then there exists $L \in M\mu_{R_1}$ and $M \in M\mu_{R_2}$ such that $a \in L, b \notin L$ and $b \in M, a \notin M$. Then $L \in M\mu_{R_1}$ and $M \in M\mu_{R_2} \Rightarrow L \cap N \in M\mu_{R_1}(N)$ and $M \cap N \in M\mu_{R_2}(N)$. Again since, $a, b \in N$ then $a \in L \cap N, b \notin L \cap N$ and $a \notin M \cap N, b \in M \cap N$. Hence N satisfy.

Lemma 3.12

Let V be a subset of a micro bitopological space U, $x \in M\mu_{R_{1,2}}cl(V)$ iff $H \cap V \neq \phi$ for every $M\mu_{R_{1,2}} -$ open set H of U containing x.

Proof

Let U be a micro bitopological space. Assume that $x \in M\mu_{R_{1,2}}cl(V)$. Suppose $H \cap V = \phi$. Then $x \in H$ and $x \notin V \Rightarrow x \notin M\mu_{R_{1,2}}cl(V)$ which is a contradiction to assumption. $H \cap V \neq \phi$ for all $x \in H$. Conversely, suppose $x \notin M\mu_{R_{1,2}}cl(V)$. Then $x \notin V$. Our assumption $x \notin H \cap V \Rightarrow x \notin H$. Therefore, $H \cap V = \phi$ which is a contradiction. Therefore, $x \in M\mu_{R_{1,2}}cl(V)$.

Definition 3.13

Let $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ be a micro bitopological space. A subset A of U is said to be micro generalized closed $(M\mu_{R_{1,2}} - g.cl)$ if $M\mu_{R_{1,2}}cl(A) \subseteq L$ where L is $M\mu_{R_{1,2}}$ open and $A \subseteq L$.

Definition 3.14

A micro bitopological space $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is said to be $M\mu_{R_{1,2}} - T_{1/2}$ space if every micro generalized closed space is $M\mu_{R_{1,2}}$ closed.

Theorem 3.15

Let U be a micro bitopological space. A subset A of U is $M\mu_{R_{1,2}} - g.cl$ iff $M\mu_{R_{1,2}}cl(x) \cap A \neq \phi$ for every $x \in M\mu_{R_{1,2}}cl(A)$.

Proof

To prove A is $M\mu_{R_{1,2}} - g.cl$. Let H be a $M\mu_{R_{1,2}}$ open set such that $A \subseteq H$. Let $x \in M\mu_{R_{1,2}}cl(A)$. Then we take $z \in M\mu_{R_{1,2}}cl(x)$ and $z \in A \subseteq H$. Then by above lemma 3.12, $H \cap x \neq \phi, x \in H$. Therefore, $M\mu_{R_{1,2}}cl(A) \subseteq H$.

A is $M\mu_{R_{1,2}} - g.cl$. Conversely, Let A be $M\mu_{R_{1,2}} - g.cl$ subset of U and $x \in M\mu_{R_{1,2}}cl(A)$.

To prove $M\mu_{R_{1,2}}cl(x) \cap A \neq \phi$. Suppose, $M\mu_{R_{1,2}}cl(x) \cap A = \phi$. Since, $M\mu_{R_{1,2}}cl(x)$ is $M\mu_{R_{1,2}}$ closed. Then $U - M\mu_{R_{1,2}}cl(x)$ is $M\mu_{R_{1,2}}$ open. $A \subseteq U - M\mu_{R_{1,2}}cl(x)$. Therefore $x \notin M\mu_{R_{1,2}}cl(A)$. This is a contradiction. Therefore $M\mu_{R_{1,2}}cl(x) \cap A \neq \phi$.

Theorem 3.16

Let H be a $M\mu_{R_{1,2}} - g.cl$ subset of a micro bitopological space U and $H \subseteq V \subseteq M\mu_{R_{1,2}}cl(H)$. Then V is also $M\mu_{R_{1,2}} - g.cl$.

Proof

Given $H \subseteq V \subseteq M\mu_{R_{1,2}}cl(H)$. Let S be a $M\mu_{R_{1,2}}$ open subset of U. Then by given, $H \subseteq V \subseteq S$ and H is $M\mu_{R_{1,2}} - g.cl$ then $M\mu_{R_{1,2}}cl(H) \subseteq S \Rightarrow M\mu_{R_{1,2}}cl(H) \subseteq M\mu_{R_{1,2}}cl(V) \subseteq M\mu_{R_{1,2}}cl\,(M\mu_{R_{1,2}}cl(H)) = M\mu_{R_{1,2}}cl(H)$. $M\mu_{R_{1,2}}cl(V) \subseteq M\mu_{R_{1,2}}cl(H) \subseteq S$. $M\mu_{R_{1,2}}cl(V) \subseteq S$. Hence V is $M\mu_{R_{1,2}} - g.cl$.

Theorem 3.17

Let U be a micro bitopological space and $x \in U$, either $\{x\}$ is $M\mu_{R_{1,2}}$ closed or $U - \{x\}$ is $M\mu_{R_{1,2}} - g.cl$.

Proof

Let take $\{x\}$ is not $M\mu_{R_{1,2}}$ closed set then $\{x\}$ is $M\mu_{R_{1,2}}$ open set. Then $U-\{x\}$ is not $M\mu_{R_{1,2}}$ open set. Let H be a $M\mu_{R_{1,2}}$ open set such that $U-\{x\} \subseteq H$. Then, $U=H$. Hence $M\mu_{R_{1,2}}cl(U-\{x\}) \subseteq H$. Therefore, $U-\{x\}$ is $M\mu_{R_{1,2}}-g.cl$.

Theorem 3.18

Let U be a micro bitopological space. U is micro symmetric iff $\{x\}$ is $M\mu_{R_{1,2}}-g.cl$ for each $x \in U$.

Proof

Assume that U is $M\mu_{R_{1,2}}-$ symmetric. Let $x \in H$ is $M\mu_{R_{1,2}}$ open set, but $M\mu_{R_{1,2}}cl(x)$ not a subset of H. Then $M\mu_{R_{1,2}}cl(x) \subseteq U-H$. Then $M\mu_{R_{1,2}}cl(x) \cap (U-H) \neq \boldsymbol{\phi}$. Now, take $y \in M\mu_{R_{1,2}}cl(x) \cap (U-H)$ by assumption $y \in M\mu_{R_{1,2}}cl(x)$ then $\{x\} \in M\mu_{R_{1,2}}cl(y) \subseteq U-H$. Therefore, $\{x\} \notin H$ this is a contradiction.

$\{x\}$ is $M\mu_{R_{1,2}}-g.cl$. Conversely, Assume that $\{x\}$ is $M\mu_{R_{1,2}}-g.cl$. To prove, U is $M\mu_{R_{1,2}}-$ symmetric. Suppose$U$ is not $M\mu_{R_{1,2}}-$ symmetric. Let $x \in M\mu_{R_{1,2}}cl(y)$ but $y \notin M\mu_{R_{1,2}}cl(x)$. The $y \in U-M\mu_{R_{1,2}}cl(x)$ and by assumption, $M\mu_{R_{1,2}}cl(y) \subseteq U-M\mu_{R_{1,2}}cl(x)$. Then $x \in U-M\mu_{R_{1,2}}cl(x)$ which is a contradiction.

U is micro symmetric.

Definition 3.19

A micro bitopological space U is said to be micro bitopological regular space if for every $u \in U$ and a $M\mu_{R_2}$ closed set H such that $u \notin H$ then there exists a $M\mu_{R_2}$ open set V and $M\mu_{R_1}$ open set W such that $u \in V$ and $H \subseteq W$ and $V \cap W = \boldsymbol{\phi}$.

Definition 3.20

A micro bitopological space U is said to be micro bitopological normal space if a $M\mu_{R_1}$ closed set H and $M\mu_{R_2}$ closed set V with $H \cap V = \boldsymbol{\phi}$ then there exists a $M\mu_{R_1}$ open set P and $M\mu_{R_2}$ open set Q such that $H \subseteq P, V \subseteq Q$and $P \cap Q = \boldsymbol{\phi}$.

4. MICRO BITOPOLOGICAL DIFFERENCE SET

Definition 4.1

A micro bitopological space U contain a subset K is called micro bitopological difference set $(M\mu_{R_{1,2}}-D)$ if there exists $M\mu_{R_{1,2}}$ open sets H and V of U such that $H \neq U$ and $K = H-V$.

Example 4.2

Consider the example 3.4, Let $s \in U$. Take a $M\mu_{R_{1,2}}$ open set $H=\{s,u\}$ and $V=\{u]$ then $A=H-V=\{s\}$.

A is $M\mu_{R_{1,2}}-D$ open set. We say that, every $M\mu_{R_{1,2}}$ open sets are $M\mu_{R_{1,2}}-D$ open sets. The converse is need not true from the above example.

Definition 4.3

A subset A is said to be $M\mu_{R_{1,2}}-D_0$ space if for any pair of distinct elements $h, v \in U$ there exists a $M\mu_{R_{1,2}}-D$ set H of U such that $h \in H, v \notin H$ or $v \in H, h \notin H$.

Definition 4.4

A subset A is said to be $M\mu_{R_{1,2}}-D_1$ space if for any pair of distinct elements $h, v \in U$ and there exists two $M\mu_{R_{1,2}}-D$ set H and V such that $h \in H, v \notin H$ and $v \in V, h \notin V$.

Definition 4.5

A subset A is said to be $M\mu_{R_{1,2}}-D_2$ space if for any pair of distinct elements $h, v \in U$ and there exists two disjoint $M\mu_{R_{1,2}}-D$ set H and V containing h and v respectively.

Note 4.6

A micro bitopological space the following properties are holds:

(1) If $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}}-T_i$ space, then $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}}-D_i$, $i=0,1,2$.

(2) If $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}}-D_i$ space, then $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}}-D_{i-1}, i=0,1,2$.

Theorem 4.7

A micro bitopological space the following properties are equivalent:

(1) $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}} - T_0$.

(2) $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}} - D_0$.

Proof

$(1) \Rightarrow (2)$ Its proved from the above note 4.6.

$(2) \Rightarrow (1)$ Let $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}} - D_0$. Let $h, v \in U$, then atleast one of them, say $h \in K, v \notin K$ Where K is a $M\mu_{R_{1,2}} - D$ set. Suppose that $K = K_1 - K_2$, where $K_1 \neq U$ and K_1, K_2 are $M\mu_{R_{1,2}}$open sets of U. The $h \in K_1, v \notin K$. Then we take two cases: (1) $v \notin K_1$, (2)$v \in K_1, v \in K_2$. In case (1) h belongs to K_1 but that does not contain v. In case (2)K_2 contains v but does not contain h because $h \in K$. Therefore $(U, \tau_{R_{1,2}}(X), \mu_{R_{1,2}}(X))$ is $M\mu_{R_{1,2}} - T_0$.

Theorem 4.8

A micro bitopological space is $M\mu_{R_{1,2}} -$ symetric space. If the space is $M\mu_{R_{1,2}} - T_0$ then its $M\mu_{R_{1,2}} - D_1$.

Proof

Let $h, v \in U$ and $h \neq v$. By given, $M\mu_{R_{1,2}} - T_0$ then $h \in U_1 \subseteq U - \{v\}$ where U_1 is $M\mu_{R_{1,2}} -$ open set. Then $h \notin M\mu_{R_{1,2}}cl\{v\}$ and by the symmetric part, $v \notin M\mu_{R_{1,2}}cl\{h\}$. Then there exists a $M\mu_{R_{1,2}} -$ open set U_2 such that $v \in U_2 \subseteq U - \{h\}$. Since every $M\mu_{R_{1,2}}$ open set is $M\mu_{R_{1,2}} - D$ set, and this satisfy the $M\mu_{R_{1,2}} - D_1$.

Conclusions

In this paper we introduced different types of separation axioms on a micro bitopological space. We identify Kernel for micro bitopological space in the future.

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