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Network Analysis from the Edge Binding Number for Coprime graph of Groups

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ABSTRACT

Binding number is used to quantify the network's vulnerability as a parameter from the viewpoint of neighborhood structure. A graph Γ represents the network, and min $\left\{\frac{N(T)}{T}: T \subseteq E(G)\right\}$ is the Edge binding number **EBN** of Γ. Let G be a vertex set, where G is a finite group with identity e. If two distinct vertices a and b are connected if and only if $(|a|, |b|) = 1$. This is called the coprime graph of G and its denoted by Γ_G. In this paper, we investigate the exact **EBN** for the network settings that are constructed as Γ_G and discussed its bound range.

Keywords: *Network; Binding number; Coprime graph.*

1. INTRODUCTION

In recent years, network attacks have increased in frequently. Network attacks refer to unauthorized and malicious activities targeting computer network, systems and infrastructure. These attacks aim to compromise network security, steal sensitive data, disrupt service, or gain unauthorized access.

The globe has seen some well-known cyber-attack:

- Yahoo! warned the public in September 2016 that a cyber-attack in 2014 hacked 500 million user accounts. The attack was thought to have been carried out by a state-sponsored actor. Three months later, the business announced that it had found another breach that had taken place in August 2013. It was one of the worst data breaches ever, with Yahoo! estimating that 1 billion user accounts had been compromised. All 3 billion Yahoo! accounts were found to have been accessed when the FBI became involved, making it an unprecedented breach.
- The Unique Identification Authority of India created Aadhaar, the largest ID database in the world, in 2009. A 12-digit unique identification number, fingerprint scans of all ten fingers, two iris scans, name, gender, and contact details were among the details of almost 1.1 billion Indian individuals that were included in the database. Applying for state aid or financial assistance, purchasing a cellular SIM card, opening a bank account, enrolling in utilities, and completing other administrative tasks all require the card. The Aadhaar database hack was reported in January 2018, making it one of the largest data breaches of the year.

Analyzing the neighborhood structure of the network, beginning with the weakest node, or identifying and obtaining the most important nodes of the entire network, then launching intense attacks on these crucial components, are two examples of successful network attack techniques. Given these facts and contexts, network security and design have provided the evaluation of network vulnerability a lot of attention[see [2], [5], [6], [7] and [9] for more details].

Edge binding number (**EBN**) $\mathbf{b_1}$ of a graph is study of minimizing $\frac{N(T)}{T}$, where the T is subset of $E(G)$ and neighbourhood of the set is $N(T)$. If we think the greater binding number (**BN**) is better, then a network with a high binding number is actually quite dense, with many sites connected. This indicates that building channels requires a significant amount of financial, material, and human resources, which raises the network's overall cost. Therefore we have to find minimum binding number. This article's goals are to provide exact binding number based on graph analysis, give network designers a precise and useful reference, and create a standard for network design based on network metrics such as binding number. The type of security criteria that the built network must meet, as well as the bind number conditions that must be met to guarantee the network's resilience and vulnerability.

Algebraic graph theory and group theory are closely related to one another. Algebraic graphs can be constructed in many ways, some special graphs are Finite symmetric graph, Cayley graph, etc. Liu et. al. [10] initially proposed a concept of a graph's $\mathbf{b_1}$ in 2001. He provided **BN** of a few plane graphs. The **BN** of Γ_G is less than or equal to 1, as discussed in the earlier paper [1]. We investigate the **EBN** of the Γ_G based on the previous research finding.

The coprime graph was created in 1997 by Paul Erdos and N. Sarkozy [11]. In 2016, the non-coprime graph was introduced by F.Mansoori, et al [3]. Let G be the identity finite group. The graph with Gas the vertex set is the coprime of G. Adjacent exists in Γ_G, whenever order of two different vertices are relatively coprime.

The following definition is for graph terminology [4]. The network is represented by the graph model $\Gamma = (V, E)$ from the perspective of graph theory, where the set of channels is represented by the edge set $E = E(\Gamma)$, and the site set is represented by the vertex set $V = V(\Gamma)$. A graph is referred to be a complete graph K_n if it has n vertices connected to one another. Let $S \subseteq V(\Gamma)$. In Γ, $N(c)$ is the open neighborhood of a such that $N(c) = \{d \in V(\Gamma) | (c, d) \in E(\Gamma)\}$. The open neighborhood $N(S) = \bigcup_{u \in T} N(u)$

The next part makes use of the definition and outcomes that follow.

Definition 1.2 [10] The Edge binding number **EBN** of a graph Γ is defined by, $b_1(\Gamma) = min\left\{\frac{|N(T)|}{|T|} : T \subseteq E(\Gamma) \,\&\, N(T) \neq E(\Gamma)\right\}$.

One can refer [8] for the group terminology. All groups are considered as finite in this paper. The number of elements of G is called its order and is denoted by $|G|$. The order of an element x of G is the smallest positive integer n such that $x^n = e$. The order of an element x is denoted by $|x|$. The set $\mathbb{Z}_n = \{0,1,2, \dots, n-1\}$ for $n \geq 1$ is a group under addition modulo n. Dihedral Group D_{2n} is a group generated by two elements a, b, $D_{2n} = \{a, b | a^n = e, b^2 = e, bab^{-1} = a^{-1}\}$. Dicyclic Group $Dic_{4n}(n > 1)$ is an extension of the cyclic group of order two by a cyclic group of order $2n$, $D_{4n} = \{a, b | a^{2n} = e, b^2 = a^n, b^{-1}ab = a^{-1}\}$. The set of all permutation of n symbols is called the Symmetric Group S_n of degree n with order is $n!$. The group of even permutation of S_n is denoted by A_n and is called the Alternating Group of degree n. Also A_n has $\frac{n!}{2}$ elements. Here $M(n, F, +)$, $GL(n, F)$, $SL(n, F)$ denote the collection of all $n \times n$ matrices, General Linear Group and Special Linear Group respectively.

2. EDGE BINDING NUMBER

2.1 Motivation

Results for **BN** in various network configurations are provided Wang and Gao 2021 [12]. Their discussed about the bounds of **BN**.

The **BN** involves the order of the vertices and edges. So the binding is highly dependent on the order of the vertices in the graph. It inspires us to find the **EBN** for the coprime graph of groups. The objective of this section is to determine the precise value of **EBN** for the Γ_G network setup.

2.2 Main Result

Theorem 2.1 Let G be isomorphic to one of the following groups: $\mathbb{Z}_{p^n}$ and $M(n, \mathbb{Z}_{p^n}, +)$. Then $b_1(\Gamma_G) = |G| - 2$.

Proof. If $G \cong \mathbb{Z}_{p^n}$ or $M(n, \mathbb{Z}_{p^n}, +)$, then the order of the vertices of Γ_G is multiple of p. Therefore the coprime graph Γ_G will be the complete bipartite graph say, $K_{1,n}$.

Here every edges are adjacent each other. We can choose only one arbitrary edge e for S, otherwise $N(S)$ will be the $E(\Gamma_G)$.

Now $N(S)$ is $N(S)/\{e\}$, so $b_1(\Gamma_G) = |G| - 2$.

Theorem 2.2 Let G be a finite additive abelian group and $p_1, p_2, \dots, p_n$ be the distinct prime numbers. Assume that $G \cong \prod_{i=1}^{n} G_i$ where $|G_i|$ is p_i-group with $|G_{i_j}| < |G_{i_{j+1}}|$ for $i_j = 1,2, \dots, n$ and $j = 1,2,3, \dots, n-1$.

Then the order of edges in Γ_G is,

$$|E(\Gamma_G)| = \sum_{k=1}^{n} 2^{n-k} \left[\sum_{i_1=1(i_1<i_j)}^{k} \prod_{j=1}^{n-(k-1)} (p_{i_j} - 1) \right].$$

Proof. Let $G \cong \prod_{i=1}^{n} \mathbb{Z}_{p_i^{\alpha_i}}$ and $\prod_{i=1}^{n} p_i = \prod_{i=1}^{n} p_{i_j}$, where $i = p_i$, where $1 \leq i \leq n$ and j is place of p.

Let $i_j = 1,2$ then $G \cong \mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}}$

Divide the vertex set $V(\Gamma_G)$ into $2^2 = 4$ vertex sets such that

$V_1 = \{v_1 : |v_1| = 1\}$

$V_2 = \{v_2 : |v_2| \equiv 0 \ (\text{mod}\ p_1)\ only\}$

$V_3 = \{v_3 : |v_3| \equiv 0 \ (\text{mod}\ p_2)\ only\}$

$V_4 = \{v_4 : |v_4| \equiv 0 \ (\text{mod}\ p_1 p_2)\ only\}$

Here $|V_1| = 1, |V_2| = p_1 - 1, |V_3| = p_2 - 1$ and $|V_4| = (p_1 - 1)(p_2 - 1)$.

From the observation, the adjacency of vertex sets are following without repeating,

V_1 is connected to all other sets and the set of all vertices in V_2 is adjacent to all the vertices in V_3 only.

Hence Γ_G is a 4-partite graph.

$|E(\Gamma_G)|$ is between the adjacent vertex sets V_i and V_j is $|V_i| \times |V_j|$.

$|E(\Gamma_G)| = |V_1| \times |V_2| + |V_1| \times |V_3| + |V_1| \times |V_4| + |V_2| \times |V_3|$

$$= 2\prod_{i=1}^{2}(p_i - 1) + \sum_{i=1}^{2}(p_i - 1)$$

$$= \sum_{k=1}^{2} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\ \prod_{j=1}^{n-(k-1)}(p_{i_j} - 1)\right]$$

Let $i_j = 1,2,3$, then $G \cong \mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}} \times \mathbb{Z}_{p_3^{\alpha_3}}$.

Divide the vertex set $V(\Gamma_G)$ into $2^3 = 8$ vertex sets such that,

$V_1 = \{v_1 : |v_1| = 1\},\ \ |V_1| = 1$

$V_2 = \{v_2 : |v_2| \equiv 0 \ (\text{mod}\ p_1)\ only\},\ \ |V_2| = p_1 - 1$

$V_3 = \{v_3 : |v_3| \equiv 0 \ (\text{mod}\ p_2)\ only\},\ \ |V_3| = p_2 - 1$

$V_4 = \{v_4 : |v_4| \equiv 0 \ (\text{mod}\ p_3)\ only\},\ \ |V_4| = p_3 - 1$

$V_5 = \{v_5 : |v_5| \equiv 0 \ (\text{mod}\ p_1 p_2)\ only\},\ \ |V_5| = (p_1 - 1)(p_2 - 1)$

$V_6 = \{v_6 : |v_6| \equiv 0 \ (\text{mod}\ p_1 p_3)\ only\},\ \ |V_6| = (p_1 - 1)(p_3 - 1)$

$V_7 = \{v_7 : |v_7| \equiv 0 \ (\text{mod}\ p_2 p_3)\ only\},\ \ |V_7| = (p_2 - 1)(p_3 - 1)$

$V_8 = \{v_8 : |v_8| \equiv 0 \ (\text{mod}\ p_1 p_2 p_3)\ only\},\ \ |V_8| = (p_1 - 1)(p_2 - 1)(p_3 - 1)$

From the observation the adjacency of vertex sets are following without repeating,

V_1 is connected to all other vertex sets. Set of all vertices in V_2 is connected to the set of all vertices in V_3, V_4, V_7 only. Set of all vertices in V_3 is connected to the set of all vertices in V_4, V_6 only and set of all vertices in V_4 is connected to the set of all vertices in V_5 only. Therefore Γ_G is a 8-partite graph.

$|E(\Gamma_G)|$ is between the adjacent vertex sets V_i and V_j is $|V_i| \times |V_j|$.

$$|E(\Gamma_G)| = \sum_{i=1}^{8}|V_1| \times |V_i| + \sum_{i=3,4,7}|V_1| \times |V_i| + \sum_{i=4,6}|V_1| \times |V_i| + |V_4| \times |V_5|$$

$$= 4\prod_{i_1=1}^{3}(p_{i_1} - 1) + 2\sum_{i_1=1(i_1<i_j)}^{2}\ \prod_{j=1}^{2}(p_{i_j} - 1) + \sum_{i_1=1}^{3}(p_{i_1} - 1)$$

$$= \sum_{k=1}^{3} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\ \prod_{j=1}^{n-(k-1)}(p_{i_j} - 1)\right]$$

Similarly we can find it for $i_j = 1,2,3, \dots, n$,

$$\sum_{k=1}^{n} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\prod_{j=1}^{n-(k-1)}(p_{i_j}-1)\right]$$

Theorem 2.3 Let G be a finite abelian group and $p_1, p_2, \ldots, p_n$ be the distinct prime numbers. Assume that $G \cong \prod G_i$ where $|G_i|$ is p_i-group with $|G_i| < |G_{i+1}|$ for $i = 1,2,\ldots,n-1$. Then

$$b_1(\Gamma_G) = \frac{\sum_{k=1}^{n} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\prod_{j=1}^{n-(k-1)}(p_{i_j}-1)\right]-1}{\sum_{k=1}^{n} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\prod_{j=1}^{n-(k-1)}(p_{i_j}-1)\right]-\sum_{i=1}^{2} p_i+1}$$

Proof. Let $G \cong \prod \mathbb{Z}_{p_i^{\alpha_i}}$.

Let $V_1, V_2, V_3 \subset V(\Gamma_G)$ such that,

$V_1 = \{a: |a| \equiv e\}$

$V_2 = \{b: |b| \equiv 0 \pmod{p_1}\ only\}$

$V_3 = \{c: |c| \equiv 0 \pmod{p_2}\ only\}$

Here $|V_1| = 1, |V_2| = p_1 - 1$ and $|V_3| = p_2 - 1$.

Without loss of generality, let $V_1 = \{v_1\}$, $V_2 = \{v_2, v_3, \ldots, v_{p_1}\}$ and

$V_3 = \{v_{p_1+1}, v_{p_1+2}, \ldots, v_{p_1+p_2-1}\}$

From the Theorem 2.2,

$$|E(\Gamma_G)| = \sum_{k=1}^{n} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\prod_{j=1}^{n-(k-1)}(p_{i_j}-1)\right]$$

Let A, B be the subsets of $E(\Gamma_G)$ such that,

$A = \{e_a: e_a \in v_1 - v_2, v_1 - v_{p_1+1}\}$ with $|K| = 2$

$B = \{e_b: e_b \in v_2 - V_3, v_{p_1+1} - V_2\}$ with $|L| = p_1 + p_2 - 1$

where $v_1 - v_2$ and $v_1 - v_{p_1+1}$ is set of all edges from v_1 to v_2 and from v_1 to v_{p_1+1} respectively; where $v_2 - V_3$ and $v_{p_1+1} - V_2$ is set of all edges from v_2 to V_3 and from v_3 to V_2 respectively.

Choose $S = E(\Gamma_G)/\{A \cup B\}$. Then the $N(S)$ is all the edges of $E(\Gamma_G)$ except an edge e_b which is between v_2 and v_{p_1+1}.

$$|S| = \sum_{k=1}^{n} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\prod_{j=1}^{n-(k-1)}(p_{i_j}-1)\right]-\sum_{i=1}^{2} p_i+1$$

$$|N(S)| = E(\Gamma_G)/\{e_b\}$$

$$|N(S)| = |E(\Gamma_G)| - 1$$

$$b_1(\Gamma_G) = \frac{\sum_{k=1}^{n} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\prod_{j=1}^{n-(k-1)}(p_{i_j}-1)\right]-1}{\sum_{k=1}^{n} 2^{n-k}\left[\sum_{i_1=1(i_1<i_j)}^{k}\prod_{j=1}^{n-(k-1)}(p_{i_j}-1)\right]-\sum_{i=1}^{2} p_i+1}$$

Theorem 2.4 Let $G \cong D_{2n}$, where n is odd. Then $b_1(\Gamma_{D_{2n}}) = \frac{n^2+n-2}{n^2-n-1}$.

Proof. Let $G \cong D_{2n}$ and $V(\Gamma_G) = \{v_1, v_2, v_3, \ldots, v_{2n}\}$

Split the vertices of $V(\Gamma_G)$ into subsets P, Q, R such that,

$P = \{v: |v| = e\}$ with $|P| = 1$

$Q = \{w: |w| = 2s,\ s \in \mathbb{N}\}$ with $|Q| = n - 1$

$R = \{x: |x| = odd\}$ with $|R| = n$

Without loss of generality, let $P = \{v_1\}, Q = \{v_2, v_3, \ldots, v_n\}$ and

$R = \{v_{n+1}, v_{n+2}, \ldots, v_{2n}\}$.

Observation from these vertex sets, Γ_G is a complete tripartite graph $K_{1,n-1,n}$.

So the order of edges in Γ_G is n^2+n-1.

Let K, L be the subsets of $E(\Gamma_G)$ such that,

$K = \{e_x : e_x \in v_1 - v_2, v_1 - v_{n+1}\}$ with $|K| = 2$

$L = \{e_y : e_y \in v_2 - R, v_{n+1} - Q\}$ with $|L| = 2n-2$

where $v_1 - v_2 and v_1 - v_{n+1}$ is set of all edges from v_1 to v_2 and from v_1 to v_{n+1} respectively; where $v_{n+1} - Q$ and $v_2 - R$ is set of all edges from v_{n+1} to Q and fromv_2 to Q respectively.

Choose $S = E(\Gamma_G)/\{K \cup L\}$, Then the $N(S)$ is all the edges of $E(\Gamma_G)$ except an edge e_y which is between v_2 and v_{n+1}. Also $|S| = n^2 - n - 1$ and $|N(S)| = n^2 + n - 2$

$$\therefore\ b_1(\Gamma_G) = \frac{n^2+n-2}{n^2-n-1}$$

Theorem 2.5 Let $G \cong Dic_{4n}$, where n is odd. Then $b_1(\Gamma_G) = \frac{2n^2+3(n-1)}{2n^2-3}$

Proof. Let $G \cong Dic_{4n}$ and

$$V(\Gamma_G) = \{v_1, v_2, v_3, \dots, v_n, v_{n+1}, \dots, v_{2n}, v_{2n+1}, \dots, v_{3n}, v_{3n+1}, \dots, v_{4n}\}.$$

Split the vertices of $V(\Gamma_G)$ into subsets P, Q, R, S such that,

$P = \{v : |v| = e\}$ with $|P| = 1$

$Q = \{w : |w| = odd\}$ with $|Q| = n-1$

$R = \{x : |x| = 2^k,\ k \in \mathbb{N}\}$ with $|R| = 2n+1$

$S = \{y : |y| = kn,\ k \in \mathbb{N}\}$ with $|S| = n-1$

Without loss of generality, let $P = \{v_1\}, Q = \{v_2, v_3, \dots, v_n\}$,

$R = \{v_{n+1}, v_{n+2}, \dots, v_{3n}, v_{3n+1}\}\ and\ S = \{v_{3n+2}, v_{3n+3}, \dots, v_{4n}\}$

Observe that, Γ_G is a 4-partite graph.

Adjacent exists from P to the sets Q, R and S only, also from$Q$ to R. So the order of edges in Γ_G is, $2n^2+3n-2$.

Let K, L be the subsets of $E(\Gamma_G)$ such that,

$K = \{e_x : e_x \in v_1 - v_2, v_1 - v_{n+1}\}$ with $|K| = 2$

$L = \{e_y : e_y \in v_2 - R, v_{n+1} - Q\}$ with $|L| = n-1+2n+1-1 = 3n-1$

where $v_1 - v_2$ and $v_1 - v_{n+1}$ is set of all edges from v_1 to v_2 and from v_1 to v_{n+1} respectively; where $v_{n+1} - Q$ and $v_2 - R$ is set of all edges from v_{n+1} to Q and from v_2 to R respectively.

Choose the set $S = E(\Gamma_G)/\{K \cup L\}$. Then the $N(S)$ is all the edges of $E(\Gamma_G)$ except one edge e_y which is between v_2 and v_{n+1}. Also $|S| = 2n^2 - 3$ and $|N(S)| = 2n^2 + 3(n-1)$.

$$b_1(\Gamma_G) = \frac{2n^2+3(n-1)}{2n^2-3}$$

Theorem 2.6 i) If $G \cong D_{2n}$. Then $b_1(\Gamma_G) = \frac{2[r(2^k+n-1)+n-1]}{2^k(2r-1)+n(2r+1)-(4r+1)}$

ii) If $G \cong Dic_{4n}$. Then $b_1(\Gamma_G) = \frac{4n(r+1)+2r(2^{k+1}-1)-2}{(2n-1)(2r+1)+2^{k+1}(2r-1)-2r}$

where $n = 2^k(2r+1)$.

Proof. Let $G \cong D_{2n}$ or Dic_{4n}.

Split the vertices of $V(\Gamma_G)$ into the subsets A, B, C, D such that,

$A = \{w : |v| = e\}$

$B = \{x : |x| = 2r+1\}$

$C = \{y : |y| = 2^k\}$

$D = \{z : |z| = l(2r+1)\}$

Observe that, Γ_G is a 4-partite graph.

Adjacent exists from A to B, C, D and from B to C. So the order of edges in Γ_G is,

$$|E(\Gamma_G)| = [|A| \times |B|] + [|A| \times |C|] + [|A| \times |D|] + [|B| \times |C|]$$

Case1: $G \cong D_{2n}$

Here $|A| = 1, |B| = 2r, |C| = n + 2^k - 1$ and $|R| = n - 2^k - 2r$

$|E(\Gamma_G)| = 2[r(2^k + n - 1) + n] - 1.$

Let $V(\Gamma_G) = \{v_1, v_2, v_3, \dots, v_n, v_{n+1}, \dots, v_{2n}\}$

Without loss of generality, let $A = \{v_1\}, B = \{v_2, v_3, \dots, v_{2r+1}\}$, $C = \{v_{2r+2}, v_{2r+3}, \dots, v_{n+2^k+2r}\}$ and $D = \{v_{n+2^k+2r+1}, v_{n+2^k+2r+2}, \dots, v_{2n}\}$

Let K, L be the subsets of $E(\Gamma_G)$ such that,

$K = \{e_x : e_x \in v_{2r+2} - B, v_2 - C\}$

$L = \{e_y : e_y \in v_1 - v_{2r+2},\ v_1 - v_2\}$

where $v_{2r+1} - B$ and $v_2 - C$ is set of all edges from v_{2r+2} to B and from v_2 to C respectively; where $v_1 - v_2$ and $v_1 - v_{2r+2}$ is set of all from edges from v_1 to v_2 and from v_1 to v_n respectively.

From the definition of coprime graph, the order of edges between two vertex set is the multiple of order of their vertex sets. Also one edge is becomes two times in the vertex set K which is between v_2 and v_{2r+2}.

Now,

$|K| = 2r + n + 2^k - 2$

$|K| + |L| = n + 2r + 2^k$

Choose the set $S = E(\Gamma_G)/\{K \cup L\}$. Then the set $N(S)$ is all the edges of $E(\Gamma_G)$ except one edge e_x which is between v_2 and v_n. Also $|S| = 2^k(2r - 1) + n(2r + 1) - (4r + 1)$ and $|N(S)| = 2[r(2^k + n - 1) + n - 1]$.

$$b_1(\Gamma_G) = \frac{2[r(2^k + n - 1) + n - 1]}{2^k(2r - 1) + n(2r + 1) - (4r + 1)}$$

Case2: $G \cong D_{2n}$

Here $|A| = 1, |B| = 2r, |C| = 2n + 2^{k+1} - 1$ and $|D| = 2(n - 2^k - r)$.

So $|E(\Gamma_G)| = 4n(r + 1) + 2r(2^{k+1} - 1) - 1$.

Let $V(\Gamma_G) = \{v_1, v_2, v_3, \dots, v_n, v_{n+1}, \dots, v_{2n}, v_{2n+1}, \dots, v_{3n}, v_{3n+1}, \dots, v_{4n}\}$.

Without loss of generality, let $A = \{v_1\}, B = \{v_2, v_3, \dots, v_{2r+1}\}$,

$C = \{v_{2r+2}, v_{2r+3}, \dots, v_{2(r+n+2^k)}\}$ and $D = \{v_{2(r+n+2^k)+1}, v_{2(r+n+2^k)+2}, \dots, v_{4n}\}$

Let K, L be the subsets of $E(\Gamma_G)$ such that,

$$K = \{e_x : e_x \in v_{2r+1} - B,\ v_2 - C\}$$
$$L = \{e_y : e_y \in v_1 - v_{2r+1},\ v_1 - v_2\}$$

where $v_{2r+1} - B$ and $v_2 - C$ is set of all edges from v_{2r+1} to B and from v_2 to C respectively; where $v_1 - v_{2r+1}$ and $v_1 - v_2$ is set of all edges from v_1 to v_{2r+1} and from v_1 to v_2 respectively.

From the definition of corpime graph, the edges between two vertex set is multiple of the order of their vertex sets. Also one edge is becomes two times in the vertex set K which is between v_2 and v_{2r+1}.

Now,

$|K| = 2r + 2n + 2^{k+1} - 2$

$|K| + |L| = 2(r + n + 2^k)$

Choose the set $S = E(\Gamma_G)/\{K \cup L\}$. Then the set $N(S)$ is all the edges of $E(\Gamma_G)$ except one edge e_x which is between v_2 and v_{2r+1}. Also $|S| = (2n - 1)(2r + 1) + 2^{k+1}(2r - 1) - 2r$, and $|N(S)| = 4n(r + 1) + 2r(2^(k + 1) - 1) - 2$

$$b_1(\Gamma_G) = \frac{4n(r + 1) + 2r(2^{k+1} - 1) - 2}{(2n - 1)(2r + 1) + 2^{k+1}(2r - 1) - 2r}$$

Remark 2.1 Observe that for $G \cong S_n$.

Divide the set $V(\Gamma_G)$ as V_1, V_2, V_3, V_4 such that,

$V_1 = \{v_1 = |v_1| = \{e\}\}$

$V_2 = \{v_2 = |v_2| = 2r + 1,\ r \in \mathbb{N}\}$

$V_3 = \{v_3 = |v_3| = 2^k,\ k \in \mathbb{N}\}$

$V_4 = \{v_4 = |v_4| = l(2r + 1),\ l > 1,\ l, r \in \mathbb{N}\}$

Adjacent exists from V_1 to V_2, V_3, V_4 and from V_2 to V_3 for any S_n. Also adjacent exists from V_2 to V_4 for some groups.

Let T be the subset of $E(\Gamma_G)$ which contains the edges from x_1 to V_3, from x_2 to V_2 and from x_3 to $x_1 \& x_2$, where $x_1 \in V_2,\ x_2 \in V_3,\ x_3 \in V_1$.

Choose S as the set $E(\Gamma_G)$ except the vertices of T. Then $|S| = |E(\Gamma_G)| - |T|$.

Now the $N(S)$ will be $E(\Gamma_G)$ except one edge x_1, x_2, where $x_1 \in V_2,\ x_2 \in V_3$. Hence $|N(S)| = |E(\Gamma_G)| - 1$.

$$b_1(\Gamma_G) = \frac{|E(\Gamma_G)| - 1}{|E(\Gamma_G)| - |T|}$$

Theorem 2.7 Let G be a group of all $n \times n$ matrix under over $\mathbb{Z}_p$ ring of integer modulo p, p is a prime number. Then $1 < b_1(\Gamma_G) \leq 2$

Proof. Let $G \cong GL(n, \mathbb{Z}_p), SL(n, \mathbb{Z}_p)$.

Case1: Let p and n is equal to 2

Let k_1, k_2, k_3 be there subsets of $V(\Gamma_G)$ such that $\bigcup_{i=1}^{n} k_i = V\,(\Gamma_G)$ with

$K_1 = \{e\},\ |K_1| = 1$

$K_2 = \{t: |t| = 3\},\ |K_2| = 2$

$K_3 = \{s: |s| = 2\},\ |K_3| = 3$

Let $V(\Gamma_G) = \{v_1, v_2, \dots, v_6\}$. Then

$K_1 = \{v_1\}$

$K_2 = \{v_2, v_3\}$

$K_3 = \{v_4, v_5, v_6\}$

From these vertex sets, the graph Γ_G will be a complete tripartite graph K_{k_1,k_2,k_3}

Order of edges in this graph is 11 say $e_1, e_2, \dots, e_{11}$

Assume that the edges e_1, e_2, e_3 are incident between k_1 and k_3, the edges e_4, e_5 are incident between k_1 and k_2, the edges $e_6, e_7, e_8, e_9, e_{10}, e_{11}$ are between k_1 and k_3.

Let $Y = \{e_1, e_4, e_6, e_7, e_8, e_9\}$ be a subsets of $V(\Gamma_G)$,

where $e_1 = v_1v_2, e_4 = v_1v_4, e_6 = v_2v_4, e_7 = v_3v_4, e_6 = v_2v_4, e_8 = v_2v_5, e_9 = v_2v_6$.

Choose S as all edges except the set of all edges in Y. Then $|S| = 5$.

According to the Γ_G.

$$N(S) = \{e_1, e_2, e_3, e_4, e_5, e_7, e_8, e_9, e_{10}, e_{11}\}$$

$|N(S)| = 10$

$b_1(\Gamma_G) = 2$

Case2: $p > 2,\ n \geq 2$

Divide the set $V(\Gamma_G)$ as K_1, K_2, K_3, K_4 such that,

$K_1 = \{e\}$

$K_2 = \{t: |t| = 2h + 1,\ h \in \mathbb{N}\}$

$K_3 = \{s: |s| = 2^k,\ k \in \mathbb{N}\}$

$K_4 = \{u: |u| = j(2h + 1),\ j, h \in \mathbb{N},\ j > 1\}$

Adjacent exists from K_1 to K_2, K_3, K_4 and from K_2 to K_3 for any G. Also adjacent exists from K_2 to K_4 for some groups.

Let $e_1 = v_1 t, e_2 = v_1 s, t \in K_2$ and $s \in K_3$.

Let Z be the edge set which contains the edges incident on t and also incident on s.

Choose $S = E(\Gamma_G)/Z'$, where $Z' = \{e_1, e_2\} \cup Z$. Then from $\Gamma_G, |S| > \frac{|E(\Gamma_G)|}{2}$.

If we add one more vertex in S, then the $N(S)$ will be $E(\Gamma_G)$ which is a contradiction.

For this $S, N(S)$ is all edges except one edge e which is between t and s.

$$N(S) = E(\Gamma_G)/\{e\}$$

Observe that $1 < \frac{|N(S)|}{|S|} \leq 2$, otherwise $|N(S)| = E(\Gamma_G)$.

Hence $1 < b_1(\Gamma_G) \leq 2$.

Lemma 2.1 Let G_1, G_2 be the groups which are finite. If $G_1 \cong G_2$,

then $b_1(\Gamma_{G_1}) = b_1(\Gamma_{G_2})$.

Proof. Assume that $G_1 \cong G_2$. Then order of the groups and order of the elements in the groups are equal to each other. There exists the Γ_{G_1} and Γ_{G_2} be the same graph. Hence $b_1(\Gamma_{G_1}) = b_1(\Gamma_{G_2})$.

Theorem 2.8 If $G \cong \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_3, S_3, GL(2, \mathbb{Z}_2)$ or $SL(2, \mathbb{Z}_2)$, then $b_1(\Gamma_G) = 2$.

Proof. Assume that $G \cong \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_3, S_3, GL(2, \mathbb{Z}_2)\ or\ SL(2, \mathbb{Z}_2)$

Here $\mathbb{Z}_4 \cong S_3$

By the Theorem 2.1, 2.7 and Lemma 2.1,

$$b_1(\Gamma_{\mathbb{Z}_4}) = b_1(\Gamma_{GL(2,\mathbb{Z}_2)}) = b_1(\Gamma_{SL(2,\mathbb{Z}_2)}) = b_1(\Gamma_{GL(2,\mathbb{Z}_2)}) = b_1(\Gamma_{S_3}) = 2$$

Remark 2.2 Let $G \cong \mathbb{Z}_3$. Then there exists only two edges in the graph Γ_G.

Hence $b_1(\Gamma_G) = 1$.

Theorem 2.9 Consider that the G group is isomorphic to one of the following:

$D_{2n}, Dic_{4n}, S_n, A_m, SL(n, \mathbb{Z}_p)$ or $GL(n, \mathbb{Z}_p)$. Then $1 \leq b_1(\Gamma_G) \leq 2$.

Proof: The proof is followed from the Theorem 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 and Remark 2.2.

3. DISCUSSION AND CONCLUSION

The binding number is a crucial metric for assessing network performance and is frequently used to gauge the network's susceptibility. In particular, there are three primary conclusions in this article:

1. Determine the exact binding number for the network settings that are constructed as coprime graphs of groups (see Theorem 2.1, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 and Remark 2.2);
2. Determine which network setting has binding number 2 (see Theorem 2.8);
3. Present the binding number is between 1 and 2 for the network settings that are constructed as coprime graphs of groups (see Theorem 2.9).

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Author Contribution

All authors have significant contributions to this paper.

Conflict of Interest

The authors declare no conflict of interest

Data availability

No data were used to support this study

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