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On $\mathcal{K}^{\star}$-Continuous Functions In Nano Topoloical Spaces

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ABSTRACT

This article discusses several features pertaining to contra contra g^{*}-continuous maps. The nano contra g^{*}-continuity and a few nano topological functions are examined in further detail.

Keywords: *$g^{\star}$-closed, continuous, semi-continuous, $K^{\star}$-continuous.*

1. INTRODUCTION

Throughout this paper N^{X} of U represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset Q of N^{X} U, Ncl(Q) and Nint(Q) denote the nano closure of Q and the nano interior of Q respectively. We recall the following definitions which are useful in the sequel.

This article discusses several features pertaining to contra contra g^{*}-continuous maps. The nano contra g^{*}-continuity and a few nano topological functions are examined in further detail.

Definition 1.1 [9] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in U} \{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

2. The upper approximation of x with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}$.

3. The boundary region of x with respect to R is the set of all objects, which can be classified neither as x nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 1.2 [3] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$;

Definition 1.3 *[3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 1.2, $\tau_R(X)$ satisfies the following axioms:*

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call NTS U. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 1.4 *[3] If $[\tau_R(X)]$ is the nano topology on U with respect to x, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 1.5 *[3] If N^X U with respect to x and if $Q \subseteq G$, then the nano interior of Q is defined as the union of all nano open subsets of Q and it is denoted by $Nint(Q)$. That is, Nint(Q) is the largest nano open subset of Q. The nano closure of Q is defined as the intersection of all nano closed sets containing Q and it is denoted by $Ncl(Q)$. That is, $Ncl(Q)$ is the smallest nano closed set containing Q.*

Definition 1.6 *[11] A subset C of a space U is called a nano g^*-closed set if Ncl(C) ⊆ K whenever $C \subseteq K$ and K is nano g-open in N^X of U.*

Definition 1.7 *A function f: U → V is said to be a nano contra*

1. continuous [5] if f^{-1}(K) is nano closed in U for every nano open set K of V.
2. semi-continuous [5] if f^{-1}(K) is nano semi-closed in U for every nano open set K of V.
3. α-continuous [5] if f^{-1}(K) is nano α-closed in U for every open set K of V.
4. g-continuous [12] if f^{-1}(K) is nano g-closed in U for every nano open set K of V.

Definition 1.8 *[10] A function f: U → V is said to be a nano contra*

1. sg-continuous if f^{-1}(K) is nano sg-closed in U for every nano open set K of V.
2. gs-continuous if f^{-1}(K) is nano gs-closed in U for every nano open set K of V.

Theorem 1.9 *[1]*

In a N^X space,

1. every nano closed set is nnao g^*-closed.
2. every nano g^*-closed set is nano g-closed and hence nano gs-closed.

2. ON NANO $\mathcal{K}^*$-CONTINUOUS FUNCTIONS

Definition 2.1 *A function $f: B \to B$ is called a nano $\mathcal{K}^\star$-continuous (resp. nano g^*-continuous) if $f^{-1}(Q)$ is nano g^*-closed (nano g^*-closed) U for each nano open (resp. nano closed) Q of B.*

Theorem 2.2 *In N^X space,*

1. each nano contra-continuous function is nano $\mathcal{K}^\star$-continuous.
2. each nano $\mathcal{K}^\star$-continuous function is nano contra g-continuous and nano contra gs-continuous.

Proof :

It is obvious.

Remark 2.3 *The reverses of the above theorem are not true and next examples.*

Example 2.4

1. Let U = $\{p_1, p_2, p_3\}$ with U/R(X) = $\{\{p_1\}, \{p_2, p_3\}\}$ and X = $\{p_1, p_2\}$.

Then the nano topology $\tau_R(X)$ = $\{\phi, U, \{p_1, p_2\}\}$.

Let T = $\{p_1, p_2, p_3\}$ with T/R = $\{\{p_1\}, \{p_2, p_3\}\}$ and X = $\{p_1, p_3\}$.

Then the nano topology $\tau_R(X)$ = $\{\phi, V, \{p_1, p_3\}\}$.

Let $f: B \to B$ be the identity function.

Then f is nano $\mathcal{K}^\star$-continuous function but not nano contra-continuous.

2. Let U = $\{p_1, p_2, p_3\}$ with U/R = $\{\{p_1\}, \{p_2, p_3\}\}$ and X = $\{p_1, p_3\}$.

Then the nano topology $\tau_R(X)$ = $\{\phi, U, \{p_1\}, \{p_2, p_3\}\}$.

Let T = $\{p_1, p_2, p_3\}$ with T/R = $\{\{p_1\}, \{p_2, p_3\}\}$ and X = $\{p_2\}$.

Then the nano topology $\tau_R(X)$ = $\{\phi, V, \{p_2\}\}$. Let $f: U \to T$ be the identity function.

Then f is nano contra g-continuous function and nano contra gs-continuous function but not nano $\mathcal{K}^\star$-continuous.

Remark 2.5 *The composition of two nano $\mathcal{K}^\star$-continuous functions but not nano $\mathcal{K}^\star$-continuous function.*

Example 2.6 *Let U = $\{p_1, p_2, p_3\}$ with U/R = $\{\{p_1\}, \{p_2, p_3\}\}$ and X = $\{p_1, p_2\}$.*

Then the nano topology $\tau_R(X)$ = $\{\phi, U, \{p_1, p_2\}\}$.

Let T = $\{p_1, p_2, p_3\}$ with T/R = $\{\{p_1\}, \{p_2, p_3\}\}$ and Y = $\{p_1, p_3\}$.

Then the nano topology $\tau_R(X)$ = $\{\phi, T, \{p_1, p_3\}\}$.

Let H = $\{p_1, p_2, p_3\}$ with H/R = $\{\{p_1\}, \{p_2, p_3\}\}$ and X= $\{p_2\}$.

Then the nano topology $\tau_R{}'(X)$ = $\{\phi, \mathrm{H}, \{p_2\}\}$.

Let $f : B \to K$ be the identity function and g : T→ H be the identity function.

Then f is nano $\mathcal{K}^\star$-continuous function and g is nano $\mathcal{K}^\star$-continuous function but g o f : U → H is not naao $\mathcal{K}^\star$-continuous.

Definition 2.7 *A N^X space is called a*

1. nano AC-space if every nano gs-closed set is nano g*-closed.
2. nano α_g-space if every nano αg-closed set is nano g*-closed.
3. nano KC -space if every nano gs-closed set is nano closed.

Theorem 2.8 *Let $f : B \to K$ be a function. Then the next conditions are equivalent.*

1. f is nano $\mathcal{K}^\star$-continuous.
2. The reverse image of every nano open in K is nano g^*-closed in U.
3. The reverse image of each closed set in K is nano g^*-open in U.

Proof :

(1) ⇒ (2) : Let S be any open set in K. By the assumption of (1), f^{-1}(S) is nano g^*-closed in U.

(2) ⇒ (3) : Let S be any closed set in K. Then $K - \mathrm{S}$ is open set in K. By the assumption of (2), $f^{-1}(K - \mathrm{S})$ = U $-f^{-1}$(S) is nano g^*-closed in U. Therefore f^{-1}(S) is nano g^*-open in U.

(3) ⇒ (1) : Let S be any open set in K. Then $K - \mathrm{S}$ is closed in K. By the assumption of (3), $f^{-1}(K - \mathrm{S})$ = X $-f^{-1}$(S) is nano g^*-open in U. Therefore f^{-1}(S) is nano g^*-closed in U. Hence f is nano $\mathcal{K}^{\star}$-continuous function.

Definition 2.9 *A function $f : B \to K$ is called a nano*

1. g^*-irresolute if f^{-1}(Q) is nano g^*-closed in U for every nano g^*-closed set Q of K.
2. gc-irresolute if $f^{-1}(Q)$ is nano g-closed in U for every nano g-closed set Q of K.
3. αg-irresolute if f^{-1}(Q) is nano αg-closed in U for every nano αg-closed set Q of K.
4. sg-irresolute if f^{-1}(Q) is nano sg-closed in U for every nano sg-closed set Q of K.
5. pre g^*-closed if f(Q) is nano g^*-closed in K for every nano g^*-closed set Q of U.
6. preclosed if f(Q) is preclosed in K for every nano closed set Q of U.

Theorem 2.10 *Let $f : B \to K$ be surjective, pre nano g^*-closed and nano g^*-irresolute and $f: B \to K$ be any function. Then $g\ o\ f: U \to H$ is nano $\mathcal{K}^{\star}$-continuous iff g is nano $\mathcal{K}^{\star}$-continuous.*

Proof :

Let $g\ o\ f$: U $\to H$ be nano $\mathcal{K}^{\star}$-continuous function. Let K be an open subset of H. Then $(g\ o\ f)^{-1}$(K) $= f^{-1}(g^{-1}$(K)) is a nano g^*-closed subset of U. Because f is nano pre g^*-closed, $f(f^{-1}(g^{-1}$(K))) = g^{-1}(K) is nano g^*-closed in K. Hence g is nano $\mathcal{K}^{\star}$-continuous function.

Conversely, let g: K $\to H$ be nano $\mathcal{K}^{\star}$-continuous function. Let S be an open subset of H. Since g is nano $\mathcal{K}^{\star}$-continuous, g^{-1}(G) is nano g^*-closed in K. Since f is nano g^*-irresolute, $f^{-1}(g^{-1}(S)) = (g\ o\ f)^{-1}$(S) is nano g^*-closed in U. Hence $g\ o\ f$ is nano $\mathcal{K}^{\star}$-continuous function.

Theorem 2.11 *If $f : B \to K$ is nano g^*-irresolute function and g : $K \to H$ is contra-continuous function, then the $g\ o\ f: U \to H$ is nano $\mathcal{K}^{\star}$-continuous function.*

Proof:

Let S be an nano open set in H. Since g is contra-continuous, g^{-1}(S) is nano closed in K. It implies that g^{-1}(S) is nano g^*-closed in K. Since f is nano g^*-irresolute, $f^{-1}(g^{-1}$(S)) = (g o f)$^{-1}(S)$ is nano g^*-closed in U. Therefore gof is nano $\mathcal{K}^{\star}$-continuous function.

Theorem 2.12 *If $f: B \to K$ is nano gc-irresolute function and $g: K \to H$ is nano $\mathcal{K}^{\star}$-continuous function, then go $f: U \to H$ is nano contra g-continuous function.*

Proof:

Let S be a nano open set in H. Since g is nano $\mathcal{K}^{\star}$-continuous function, $g^{-1}(S)$ is nano g^*-closed in K. It implies that nano g-closed in K. Since f is gc-irresolute, $f^{-1}(g^{-1}$(S)) = (g o f)$^{-1}$(S) is nano g-closed in U. Thus $g\ o\ f$ is nano contra g-continuous function.

Theorem 2.13 *Let $f: B \to K$ be a function and $g: U \to U \times K$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in U$. If g is nano $\mathcal{K}^{\star}$-continuous, then f is nano $\mathcal{K}^{\star}$-continuous.*

Proof:

Let A be a nano open set in K. Then $U \times A$ is a nano open set in $A \times K$. It follows from Theorem 2.8 that f^{-1}(A) $= g^{-1}(U \times A)$ is nano g^*-closed in U. Thus, f is nano $\mathcal{K}^{\star}$-continuous.

Definition 2.14 *A N^X space is called nano α_{g_1}-space if every nano g^*-closed set is nano α-closed.*

Theorem 2.15 *Let $f: B \to K$ be a nano $\mathcal{K}^\star$-continuous function. If U is nano α_{g_1}-space, then f is nano contra α-continuous function.*

Proof:

Let Q be nano open set of K. Since f is nano $\mathcal{K}^\star$-continuous, f^{-1}(Q) is a nano g^*-closed set of P. Since P is an α_{g_1}-space, $f^{-1}(Q)$ is nano α-closed set of P. Therefore f is a nano contra α-continuous function.

Theorem 2.16 *Let $f: B \to K$ be a nano contra semi-continuous function. If P is nano AC-space, then f is nano $\mathcal{K}^\star$-continuous function.*

Proof:

Let Q be a nano open set of K. Since f is nano contra semi-continuous, f^{-1}(Q) is a nano semi-closed set of P and hence nano gs-closed in P. Since U is nano AC-space, f^{-1}(Q) is a nano g^*-closed set of U. Therefore f is a nano $\mathcal{K}^\star$-continuous function.

Theorem 2.17 *Let $f: B \to K$ be a nano contra α-continuous function. If U is nano AC-space, then f is nano $\mathcal{K}^\star$-continuous function.*

Proof:

Let Q be a nano open set of K. Since f is nano conta α-continuous, f^{-1}(Q) is a nano α-closed set of U and hence nano αg-closed in Q. Since U is an AC-space, f^{-1}(Q) is a nano g^*-closed set of U. Therefore f is nano $\mathcal{K}^\star$-continuous function.

Theorem 2.18 *Let $f: B \to K$ be a nano contra gs-continuous function. If X is nano KC-space, then f is nano $\mathcal{K}^\star$-continuous function.*

Proof:

Let Q be a nano open set of K. Since f is nano contra gs-continuous, f^{-1}(Q) is nano gs-closed set of U. Since U is nano KC-space, it is a nano closed set of U. It implies that f^{-1}(Q) is nano g^*-closed set of U. Therefore f is a nano $\mathcal{K}^\star$-continuous function.

Definition 2.19 *A N^X space is called a nano locally indiscrete if every nano open set is nano closed.*

Theorem 2.20 *Let $f: B \to K$ be a surjective, nano preclosed, nano $\mathcal{K}^\star$-continuous function and U be nano KC-space, then K is nano locally indiscrete.*

Proof:

Suppose Q is nano open set in K. By hypothesis, f is nano $\mathcal{K}^\star$-continuous function, f^{-1}(Q) is nano g^*-closed and hence nano gs-closed in U. Since U is nano KC-space, f^{-1}(Q) is nano closed in U. Since f is nano preclosed, Q is nano preclosed in K. Now we have Ncl(Q) = Ncl(Nint(Q)) ⊆ Q. This means that Q is nano closed in K. Thus K is nano locally indiscrete.

Theorem 2.21 *Let U and H be N^X and K be nano KC-space. If $f: B \to K$ is nano g^*-continuous function and g: $K \to H$ is nano Cgs-continuous function, then g o f : U → H is nano $\mathcal{K}^\star$-continuous function.*

Proof:

Let S be a nano open set in H. Since g is nano contra gs-continuous, g^{-1}(S) is nano gs-closed in K. But K is nano KC-space, g^{-1}(S) is nano closed in K. Since f is nano g^*-continuous, $f^{-1}(g^{-1}$(S)) = (g o f)$^{-1}$ (S) is nano g*-closed in U. Therefore $g\ o\ f$ is nano $\mathcal{K}^\star$-continuous function.

Corollary 2.22 *Let U and H be any N^X and K be nano AC-space. If $f: B \to K$ is nano g^*-irresolute function and g: $K \to H$ is nano contra gs-continuous function, then g o f: U → H is nano $\mathcal{K}^\star$-continuous function.*

Theorem 2.23 *Let U and H be any N^X and K be a nano KC-space. If f: $B \to K$ is nano g^*-irresolute function and g: $K \to H$ is nano contra-continuous function, then g o f: U → H is nano $\mathcal{K}^\star$-continuous function.*

Proof:

Let S be a nano open set in H. Since g is nano contra-continuous, g^{-1}(S) is nano closed and hence nano gs-closed in K. But K is a nano AC-space, g^{-1}(S) is nano g^*-closed in K. Since f is nano g^*-irresolute, $f^{-1}(g^{-1}$(S)) = (g o f)$^{-1}$ (S) is nano g^*-closed in U. Therefore $go\ f$ is nano $\mathcal{K}^\star$-continuous function.

Theorem 2.24 *If $f: B \to K$ is nano $\mathcal{K}^\star$-continuous surjection and U is nano g^*-connected, then K is nano connected.*

Proof:

Suppose that K is not nano connected space. There exist non-empty disjoint nano open sets Q_1 and Q_2 such that Q = Q_1 ∪

Q_2. Therefore Q_1 and Q_2 are nano clopen in K. Since f is nano $\mathcal{K}^{\star}$-continuous, $f^{-1}(Q_1)$ and $f^{-1}(Q_2)$ are nano g^*-open in U. Moreover, $f^{-1}(Q_1)$ and $f^{-1}(Q_2)$ are non-empty disjoint and U = $f^{-1}(Q_1) \cup f^{-1}(Q_2)$. This shows that U is not nano g^*-connected. This contradicts that T is not connected assumed. Hence K is nano connected.

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