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A Network-Based Stochastic Analysis of Disease Level with Continuous Distribution

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ABSTRACT

Several blood glucose levels of a diabetic are covered in this paper. A diabetic's blood glucose level is extremely high, either because they are insulin resistant or because their body does not create enough insulin to process it.Blood glucose level of a patient with damaged pancreas may pass through fair control level and poor control level several times before it reaches controlled level during the treatment. This may lead to number of cyclesof fair control level topoor control level and then to fair control level based on the changes in the blood glucose level of the patient. Together with the expected value and variance of the number of cycles before the blood glucose level gets controlled, as well as numerical examples, the probability of creating such a number of cycles using the exponential distribution and Erlang order 2 are presented.

Keywords: Exponential distribution, ErlangPhase 2 distribution, *r-s-r* cycles, Laplace transform

1. INTRODUCTION

Diabetes is one of the largest global public health concerns, imposing a heavy global burden on public health as well as socio-economic development. The global increase in diabetes occurs because of population ageing and growth, and of increasing trends towards obesity, unhealthy diets and sedentary lifestyles. These environmental changes, superimposed on genetic predisposition, increased insulin resistance, which in concert with progressive β −cell failure, results in rising glycaemia in the non- diabetic range.Sicree R, and et. al discussed Diabetes and impaired glucose tolerance[6].Achieving glycemic control is the corner stone of any diabetes program. Tight glucose control significantly decrease the risk of developing both micro and macro vascular complications.S.K.Bhattacharya, R.Biswas, M.M.Ghosh, P.Banerjee, have studied a risk factors of diabetes mellitus [2].Glucotoxicity is defined as the non-physiologic alteration in a celluar function caused by chronic exposure to high blood glucose concentrations. Hyperglycaemia arising from any primary cause further worsens the glycaemic status. It is toxic to the β −cell; it affects the glucose sensor mechanisms where by insulin secretion in response to rising blood glucose becomes inappropriate and it impairs the glucose transporter system. The Diabetes Control and Complications Motocross Research groups have discussed the long term complications in insulin dependent diabetes mellitus [7].

All complications of organs may move to different stages during treatment, they may move to complicated stages when patient is careless andmay move to better stages when the patient has follow the things: achieve and maintain healthy body weight;be physically active – doing at least 30 minutes of regular at everyday eat a healthy diet, avoiding sugar and saturated fats; andavoid tobacco use – smoking increases the risk of diabetes and cardiovascular disease. Blood glucose level of a patient with damaged pancreas may pass through fair control level (say stage r) and poor control level (say stage s) several times,between them before the glucose level reaches controlled level due to the treatment. This may form number of cycles of fair control level to poor control level to fair control level based on the changes in the blood glucose level of the patient. We call this as one r-s-r cycle. The r-s-r cycle given here is interpreted as follows. In this transition cycle, the glucose level of the patient starts in stage-r and then move to the stage-s which is called level-1 transfer. Similarly the glucose level of the patient starts in stage-s and then move to the stage-r which is called level-2 transfer. Here a complete r-s-r cycle indicatesthe completion ofone pair of level-1 and level-2 transfers.For more details on such deficiency and disorder, one may refer to [3], [4], [5]. Using the Mathematical approach given in [1], [8], [9], and [10],we study the case of a diabetic person.

In section 2, we derive the probability generating function, mean and variance of number of cycles occur in our model during treatment time which has Exponential distribution. We present numerical examples to illustrate the applications. The assumptions of the model are given in section 2 and the results are derived there. In section 3, numerical results are discussed.In section 4, we derive the probability generating function, mean and variance of number of cycles occur in our model during treatment time which has Erlang distribution with phase 2. We present numerical examples to illustrate the applications. The assumptions of the model are given in section 4 and the results are derived there. In section 5, numerical results are discussed.

2. SECTION-2

2.1 Model Assumptions:

a) LetTbe the treatment time of the patient to bring the blood glucose level to fully controlled level which has an exponential distribution with parameter $\lambda > 0$which is denoted by $T \sim \exp(\lambda)$.

b) *Level-1*transfer in our model indicatesstarting from *stage-r*, the sugar level reaches the *stage-s*. Let level 1 transfer time be exponential with parameter α.

c) *Level-2*transfer in our model indicates starting from *stage-s*, the sugar level reaches the *stage-r*. Let level 2 transfer time be exponential with parameter β.

d) Let us assume that a complete *r-s-r* cycle indicates that after the *level-1* has been completed, the *level-2* is also ended.

2.2 Analysis:

Let*P(N)*denote the probability of N number of *r-s-r*cycles in time duration T where N is discrete variable and T is continuous variable. To study the above model the probability density function of the treatment duration timeT with parameter λ is required which is$\lambda e^{-\lambda x}$, $0 \le x < \infty$.

Case1: Probability of number of *r-s-r* cycle is 0 in duration of treatment time *T* is calculated as follows noting T has exponential distribution with parameter λ.

$$P(N=0) = \int_0^\infty e^{-\alpha t}\,\lambda e^{-\lambda t} dt + \int_0^\infty \lambda e^{-\lambda t} \int_0^t \alpha e^{-\alpha u}\, e^{-\beta(t-u)} du dt \quad (1)$$

The first term of (1) of the R.H.S is the probability that the level 1 transfer is not completed during treatment time. The second term of the R.H.S is the probability that the level 1 transfer is completedbut level-2transfer is not completed during treatment time.

$$P(N=0) = \frac{\lambda}{\alpha+\lambda} + \lambda\alpha \left(\frac{1}{\lambda+\alpha}\right)\left(\frac{1}{\lambda+\beta}\right) = \frac{\lambda}{\alpha+\lambda}\left(\frac{\alpha+\beta+\lambda}{\lambda+\beta}\right) = \lambda\left(\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right) \quad (2)$$

Case2: Probability of number of *r-s-r* cycle is 1in duration of treatment time *T* is calculated as follows when T is exponential with parameter λ.

$$P(N=1) = \int_0^\infty \lambda e^{-\lambda t} \left\{ \int_0^t g_1(u)\, e^{-\alpha(t-u)}\, du + \int_0^t g_1(u) \int_0^{t-u} \alpha e^{-\alpha v}\, e^{-\beta(t-u-v)} dv\, du \right\} dt. \quad (3)$$

Where

$$g_1(u) = \int_0^u \alpha e^{-\alpha w}\ \ \beta e^{-\beta(u-w)}\, dw. \quad (4)$$

The equation (4) marked above indicates that the pdf of one complete *r-s-r* cycle.

The first term in the R.H.S of equation (3) is the probability that the *level-1*is not finished afterone complete *r-s-r* cycle ends during treatment time.The second term is the probability that after a complete *r-s-r* cycle and a subsequent transfer to *level-1*, the nextthe *level*-2 the transferis notcompleted.

From (4)

$$g_1(u) \;= \int_0^u \alpha e^{-\alpha w} \;\; \beta e^{-\beta(u-w)}\, dw = \alpha\beta \int_0^u e^{-w(\alpha-\beta)}\, e^{-\beta u}\, dw.$$

This gives

$g_1(u) \;= \frac{\alpha\beta}{\alpha-\beta} e^{-\beta u}(1 - e^{-(\alpha-\beta)u}\,)$.(5)

We can apply the Laplace transform of pdf equation (5),

$$g_1^*(s) = \left(\frac{\alpha\beta}{\alpha-\beta}\right)\left[\frac{1}{s+\beta} - \frac{1}{s+\alpha}\right] = \frac{\alpha\beta}{(s+\alpha)(s+\beta)} \qquad (6)$$

The first term of R.H.S: of equation (3) is

$\int_0^\infty \lambda\, e^{-\lambda\, t}(g_1 * \exp(\alpha))(t)dt \;= \lambda g_1^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right)$ (7)

Similarly the second term of R.H.S: of equation (3) becomes

$$\int_0^\infty \lambda e^{-\lambda\, t}(g_1 * \alpha \exp(\alpha) * \exp(\beta))(t)dt = \lambda \int_0^\infty e^{-\lambda t}\,(g_1 * \alpha \exp(\alpha) * \exp(\beta))(t)dt.$$

$= \lambda\alpha\; g_1^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right)\left(\frac{1}{\beta+\lambda}\right).$ (8)

Using equations (6), (7) and (8), it can be seen that

$P(N=1) = \lambda g_1^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right) + \lambda\alpha\; g_1^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right)\left(\frac{1}{\beta+\lambda}\right) = \frac{\lambda g_1^*(\lambda)}{\alpha+\lambda}\left[1 + \frac{\alpha}{\beta+\lambda}\right].$

This gives

$P(N=1) = \left(\frac{\lambda}{\alpha+\lambda}\right)\left(\frac{\beta+\lambda+\alpha}{\beta+\lambda}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)\left(\frac{\beta}{\beta+\lambda}\right) = \lambda\left(\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)\left(\frac{\beta}{\beta+\lambda}\right)$ (9)

Similarly the equation for the probability of two complete *r-s-r* cycle in the treatment time is given by

$$P(N=2) = \;\lambda g_2^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right) + \lambda\alpha\; g_2^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right)\left(\frac{1}{\beta+\lambda}\right) = \frac{\lambda g_2^*(\lambda)}{\alpha+\lambda}\left[1 + \frac{\alpha}{\beta+\lambda}\right].$$

Where$g_2(u)$ is the 2 fold convolution of $g_1(u)$ with itself.

This gives

P(N = 2) $= \left(\frac{\lambda}{\alpha+\lambda}\right)\left(\frac{\beta+\lambda+\alpha}{\beta+\lambda}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^2\left(\frac{\beta}{\beta+\lambda}\right)^2 = \lambda\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^2\left(\frac{\beta}{\beta+\lambda}\right)^2$ (10)

Below is the equation for getting a complete*r-s-r* cycle of exactly k-times in the treatment time.

$P(N=k) = \left(\frac{\lambda}{\alpha+\lambda}\right)\left(\frac{\beta+\lambda+\alpha}{\beta+\lambda}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^k\left(\frac{\beta}{\beta+\lambda}\right)^k = \lambda\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^k\left(\frac{\beta}{\beta+\lambda}\right)^k$ (11)

The probability generating function of P(N=k) is $\Phi(x)=\sum_{k=0}^{\infty} P(N=k)x^k$, $0 \le x \le 1$ (12)

Using (2),(9),(10) and (11)in equation (12),

$\Phi(x) = \lambda\left(\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right) + \lambda\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)\left(\frac{\beta}{\beta+\lambda}\right)x + \lambda\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^2\left(\frac{\beta}{\beta+\lambda}\right)^2 x^2 + \cdots +$
$\lambda\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^k\left(\frac{\beta}{\beta+\lambda}\right)^k x^k + \cdots$

$\Phi(x) = \lambda\left(\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right)\left[1 - \frac{\alpha\beta}{(\alpha+\lambda)(\beta+\lambda)}\;x\right]^{-1}$(13)

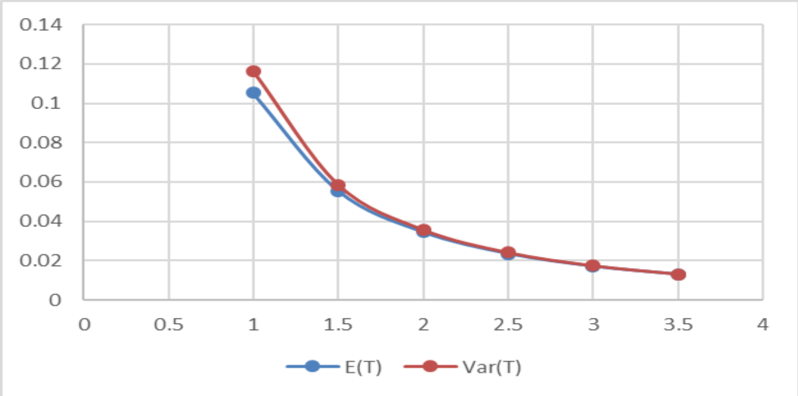
To check that $\Phi(x)$ is generating function of probabilities, we can derive $\Phi(1)=1$ by putting$x$=1. Now we can calculate the expected time to complete the *r-s-r* cycle and variance using $\Phi(x)$. It can be seen that

$\left[\frac{d\Phi}{dx}\right]_{at\;x=1} = E(N) = \frac{\alpha\beta}{\lambda(\alpha+\beta+\lambda)}$. It can be seen that $Var(N) = \Phi''(1) + E(N) - (E(N))^2$ and $\Phi''(1) = \left[\frac{2(\alpha\beta)^2}{\lambda^2(\alpha+\beta+\lambda)^2}\right]$ which gives $Var(N) = \frac{\alpha\beta[\alpha\beta+\lambda(\alpha+\beta+\lambda)]}{\lambda^2(\alpha+\beta+\lambda)^2}$.(14)

3. NUMERICAL EXAMPLES

The usefulness of the results obtained is presented in numerical examples.

Case 1: Now we fix the values of α & β and change the value of λ

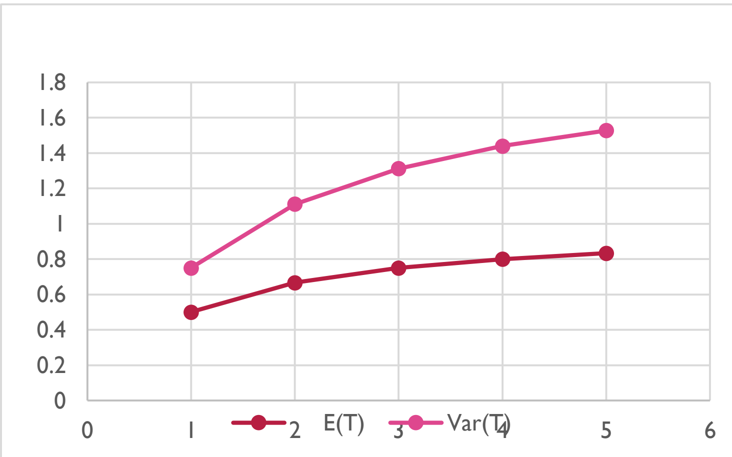


$\alpha = 0.5$	$\beta = 0.4$	
λ	E(T)	Var(T)
1	0.10526	0.11634
1.5	0.0555	0.05864
2	0.03448	0.035671
2.5	0.02353	0.02408
3	0.01709	0.01738
3.5	0.01298	0.013155

Table 1.1: Figure 1.1

We can note here that as λ increases E(T) and Var(T) decrease.

Case 2: Now we fix the values of λ & β and change the value of α

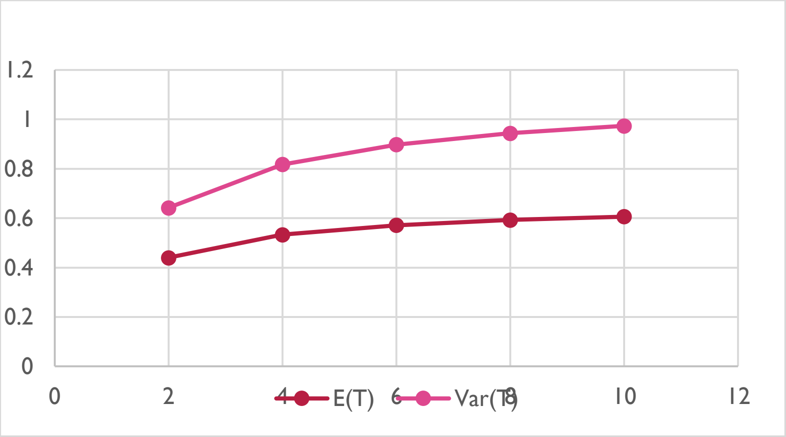


$\beta = 0.5$ $\lambda = 0.5$		
α	E(T)	Var(T)
1	0.5	0.75
2	0.6666	1.1111
3	0.75	1.3125
4	0.8	1.44
5	0.833	1.5278

Table 1.2 **Figure 1.2**

We again note here that as α increases E(T) and Var(T) also increase .

Case 3: We fix the values of α & λ then change the value of β



α = 0.4, $\lambda = 0.6$		
β	E(T)	Var(T)
2	0.44	0.64197
4	0.533	0.81777
6	0.57143	0.89795
8	0.59259	0.94376
10	0.60606	0.97337

Table 1.3 **Figure 1.3**

We can note here that as β increases E(T) and Var(T) also increase.

Finally we conclude that if λ value increases then E(T) &Var(T) decrease. Similarly if α & β values increase then E(T) and Var(T) also increase.

4. SECTION 4

In this section we study the previous model of section 2 changing the treatment time as Erlang with phase 2 and scale parameter λ .

4.1 Model Assumptions:

a) Let Tbe the treatment time of the patient to bring the blood glucose level to fully controlled level which has an Erlangdistribution with parameter λ> 0 and phase 2which is denoted by T∼E (λ, 2) whose Cdf = 1- $e^{-\lambda x}$ - λ x $e^{-\lambda x}$ and pdf = λ^2x $e^{-\lambda x}$.

b) *Level-1*transfer in our modelindicates starting from *stage-r*, the sugar level reaches the *stage-s*. Let level 1 transfer time be exponential with parameter α.

c) *Level-2*transfer in our model indicates starting from *stage-s*, the sugar level reaches the *stage-r*. Let level 2 transfer time be exponential with parameter β.

d) Let us assume that a complete *r-s-r* cycle indicates that after the *level-1* has been

completed, the *level-2* is also ended.

4.2 Analysis:

Let*P(N)*denote the probability of N number of *r-s-r*cycles in time duration T where N is discrete variable and T is E (λ, 2)random variable. To study the above model the probability density function of the treatment duration time T with E(λ, 2) is required which is $\lambda^2 x e^{-\lambda x}$, 0 ≤ *x*< ∞.

Case1: Probability of number of *r-s-r* cycle is 0 in duration of treatment time *T* is calculated as follows noting T has Erlang distribution with parameter λ and phase 2.

$$P(N=0) = \int_0^\infty e^{-\alpha t}\, \lambda^2 t e^{-\lambda t} dt + \int_0^\infty \lambda^2 t e^{-\lambda t} \int_0^t \alpha e^{-\alpha u}\, e^{-\beta(t-u)} du dt \qquad (15)$$

The first term of (1) of the R.H.S is the probability that the level 1 transfer is not completed during treatment time. The second term of the R.H.S is the probability that the level 1 transfer is completed but level-2transfer is not completed during treatment time.

$$P(N= 0) = \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) \left[\int_0^\infty e^{-\alpha t}\, e^{-\lambda t} dt + \int_0^\infty e^{-\lambda t} \int_0^t \alpha e^{-\alpha u}\, e^{-\beta(t-u)} du dt\right]$$

$$= \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) \left[\frac{1}{\alpha+\lambda} + \left(\frac{\alpha}{\alpha+\lambda}\right)\left(\frac{1}{\beta+\lambda}\right)\right] = \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right)\left[\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right] . \qquad (16)$$

Equation (16) is comparable with equation (2). The first multiplier λ is replaced by the operator$\lambda^2(-\frac{\partial}{\partial\lambda})$. The operator notation is used henceforth in this article in derivation to avoid messy expressions.

Case2: Probability of number of *r-s-r* cycle is 1 in duration of treatment time *T* is calculated as follows when T has ErlangCdf with parameter λ and phase 2.

$$P(N = 1) = \int_0^\infty \lambda^2 t e^{-\lambda t} \left\{\int_0^t g_1(u)\, e^{-\alpha(t-u)}\, du + \int_0^t g_1(u) \int_0^{t-u} \alpha e^{-\alpha v}\, e^{-\beta(t-u-v)} dv\, du\right\} dt. \quad (17)$$

Where$g_1(u) = \int_0^u \alpha e^{-\alpha w}\ \beta e^{-\beta(u-w)}\, dw$seen in (4). Using similar arguments presented in section 2, it may be seen that using (16) that

$$g_1^*(s) = \left(\frac{\alpha\beta}{\alpha-\beta}\right)\left[\frac{1}{s+\beta} - \frac{1}{s+\alpha}\right] = \frac{\alpha\beta}{(s+\alpha)(s+\beta)}$$

The first term of R.H.S: of equation (17) is

$$\int_0^\infty \lambda^2 t e^{-\lambda t} (g_1 * \exp(\alpha))(t) dt \ = \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) \left[g_1^*(\lambda) \left(\frac{1}{\alpha+\lambda}\right)\right] (18)$$

Similarly the second term of R.H.S: of equation (17) become

$$\int_0^\infty \lambda^2 t e^{-\lambda t} (g_1 * \alpha \exp(\alpha) * \exp(\beta))(t) dt = \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) \int_0^\infty e^{-\lambda t}\, (g_1 * \alpha \exp(\alpha) * \exp(\beta))(t) dt.$$

$$= \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) \alpha\, g_1^*(\lambda) \left(\frac{1}{\alpha+\lambda}\right)\left(\frac{1}{\beta+\lambda}\right). (19)$$

Using equations (17),(18) and (19), it can be seen that

$$P(N = 1) = \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) g_1^*(\lambda) \left(\frac{1}{\alpha+\lambda}\right) + \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) \alpha\, g_1^*(\lambda) \left(\frac{1}{\alpha+\lambda}\right)\left(\frac{1}{\beta+\lambda}\right)$$

$$= \lambda^2 \left(-\frac{\partial}{\partial\lambda}\right) \frac{g_1^*(\lambda)}{\alpha+\lambda}\left[1 + \frac{\alpha}{\beta+\lambda}\right].$$

This gives

$P(N=1) = \lambda^2\,(-\frac{\partial}{\partial\lambda})[\left(\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)\left(\frac{\beta}{\beta+\lambda}\right)]$.(20)

Equation (20) is comparable with equation (9). Similarly the equation for the probability of two complete *r-s-r* cycle in the treatment time is given by

$$P(N=2) = \lambda^2(-\frac{\partial}{\partial\lambda})g_2^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right) + \lambda^2(-\frac{\partial}{\partial\lambda})\alpha\, g_2^*(\lambda)\left(\frac{1}{\alpha+\lambda}\right)\left(\frac{1}{\beta+\lambda}\right) = \lambda^2(-\frac{\partial}{\partial\lambda})\frac{g_2^*(\lambda)}{\alpha+\lambda}\left[1+\frac{\alpha}{\beta+\lambda}\right].$$

where$g_2(u)$ is the 2 fold convolution of $g_1(u)$ with itself. This gives

P(N = 2) $= \lambda^2(-\frac{\partial}{\partial\lambda})\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^2\left(\frac{\beta}{\beta+\lambda}\right)^2$. (21)

Below is the equation for the probability of complete*r-s-r* cycle of exactly k-times in the treatment time.

$P(N=k) = \lambda^2(-\frac{\partial}{\partial\lambda})\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^k\left(\frac{\beta}{\beta+\lambda}\right)^k$. (22)

The probability generating function for the number of *r-s-r* cycle is

$\Phi(x)=\sum_{k=0}^{\infty} P(N=k)x^k,\ 0 \le x \le 1$ (23)

Using (16),(20),(21) and (22) in equation (23),

$$\Phi(x)= \lambda^2\,(-\frac{\partial}{\partial\lambda})[\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}] + \lambda^2\,(-\frac{\partial}{\partial\lambda})[\left(\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)\left(\frac{\beta}{\beta+\lambda}\right)]x +$$

$$\lambda^2(-\frac{\partial}{\partial\lambda})\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^2\left(\frac{\beta}{\beta+\lambda}\right)^2 x^2 + \cdots + \lambda^2(-\frac{\partial}{\partial\lambda})\left(\frac{\beta+\lambda+\alpha}{(\alpha+\lambda)(\beta+\lambda)}\right)\left(\frac{\alpha}{\alpha+\lambda}\right)^k\left(\frac{\beta}{\beta+\lambda}\right)^k x^k + \cdots$$

$\Phi(x)= \lambda^2\,(-\frac{\partial}{\partial\lambda})\,\{\left(\frac{\alpha+\beta+\lambda}{(\alpha+\lambda)(\beta+\lambda)}\right)\left[1-\frac{\alpha\beta}{(\alpha+\lambda)(\beta+\lambda)}\,x\right]^{-1}\}$ (24)

To check that Φ(x) is generating function of probabilities we can derive Φ (1) =1. Byputting$x$=1, it can be seen Φ(1) $=\lambda^2\left(-\frac{\partial}{\partial\lambda}\right)\left(\frac{1}{\lambda}\right) = 1$. Now we can calculate the expected time to complete the *r-s-r* cycle and variance using Φ(*x*). It can be seen that

$\left[\frac{d\Phi}{dx}\right]_{at\ x=1} = E(N) = \lambda^2\,(-\frac{\partial}{\partial\lambda})[\frac{\alpha\beta}{\lambda^2(\alpha+\beta+\lambda)}] = \frac{\alpha\beta(2\alpha+2\beta+3\lambda)}{\lambda(\alpha+\beta+\lambda)^2}$.(25)

It can be seen that Var(N) = Φ"(1)+E(N) - $(E(N))^2$(26)

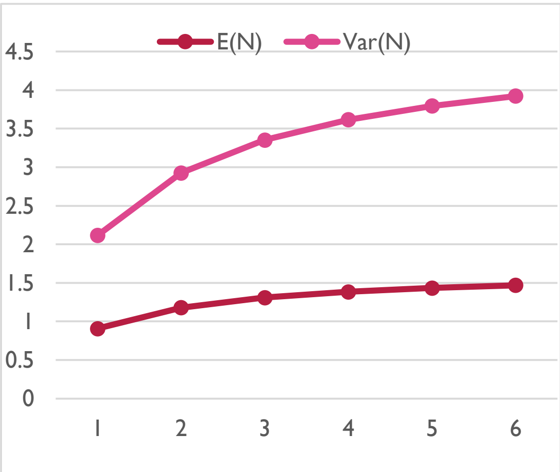
andΦ"(1) $= \lambda^2\left(-\frac{\partial}{\partial\lambda}\right)\left[\frac{2(\alpha\beta)^2}{\lambda^3(\alpha+\beta+\lambda)^2}\right]$. It can be seen that Φ"(1) $= \frac{2\alpha^2\beta^2(3\alpha+3\beta+5\lambda)}{\lambda^2(\alpha+\beta+\lambda)^3}$. (27)

Using (25), (26) and (27), Var(N) can be written.

5. Numerical examples

The usefulness of the results obtained is presented in numerical examples.

Case 1: Now we fix the values of λ& β then change the value of α

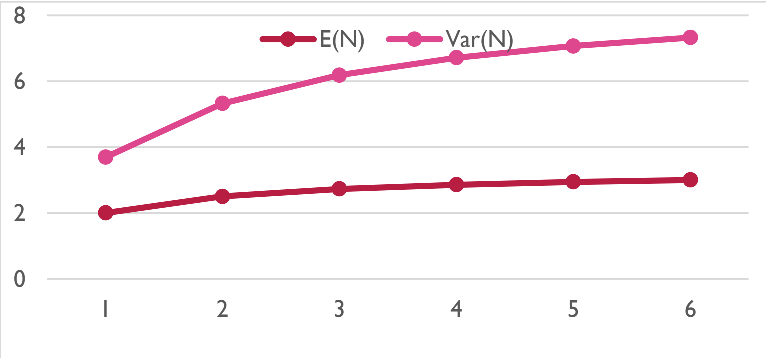


β = 0.5, λ = 0.6		
A	E(N)	Var(N)
1	0.9070294	1.2091155
2	1.179327	1.7466
3	1.308744	2.0443873
4	1.384083	2.232542
5	1.433064	2.361934
6	1.467962	2.456287

Table 2.1

We can note here that as αincreases E(T) and Var(T) also increase.

Case 2: We fix the values of α & λ then change the value ofβ

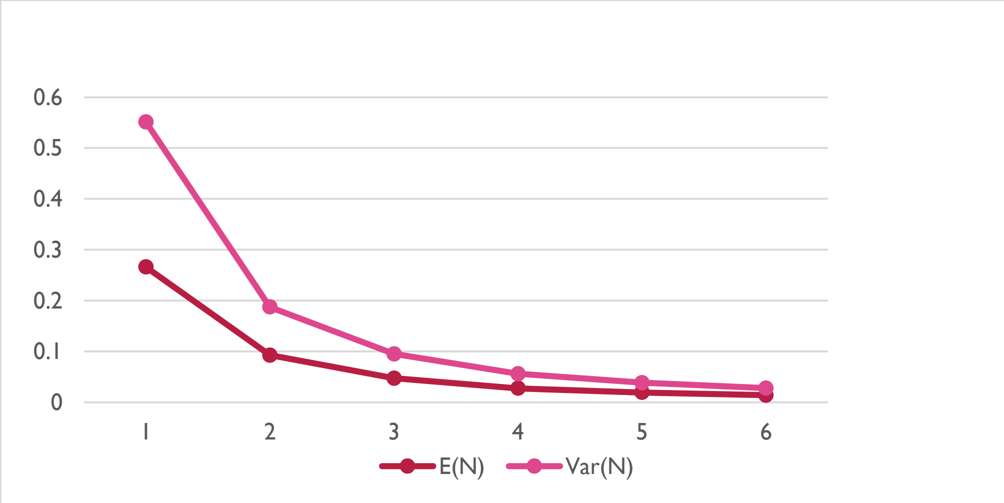


α = 0.4 , λ = 0.3		
β	E(N)	Var(N)
1	2.0061728	3.6970355
2	2.5085034	5.326701
3	2.735457	6.1872702
4	2.8645833	6.7150728
5	2.947879	7.071039
6	3.006055	7.3271056

Table 2.2

We can note here that as β increases E(T) and Var(T) also increase.

Case 3: Now we fix the values of α & β and change the value of λ



α =0.5 , β = 0.4		
λ	E(N)	Var(N)
1	0.2659279	0.2850192
2	0.0927467	0.0945593
3	0.0473372	0.0477487
4	0.0273802	0.0288768
5	0.0193047	0.0193637
6	0.0138626	0.01389166

Table 2.3

We can note here that as λ increases E(T) and Var(T) decrease.

5. CONCLUSION

Finally we conclude that if λ value increase then E(T) &Var(T) decrease. Similarly if α& β values increase then E(T) and Var(T) also increase.

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